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A methodology for building a multiregional Supply and Use Table for Italy: an updated and revised version

Renato Paniccià

Abstract

The IRPET methodology for building the multiregional Supply and Use tables for Italy has been updated in some components and significantly revised in other modules. This working paper provides a comprehensive view of what has been done both in terms of data management and balancing procedures.

JEL codes: C67, R15, D57

Keywords: Regional Economics, Input-Output Tables, Balancing Algorithms

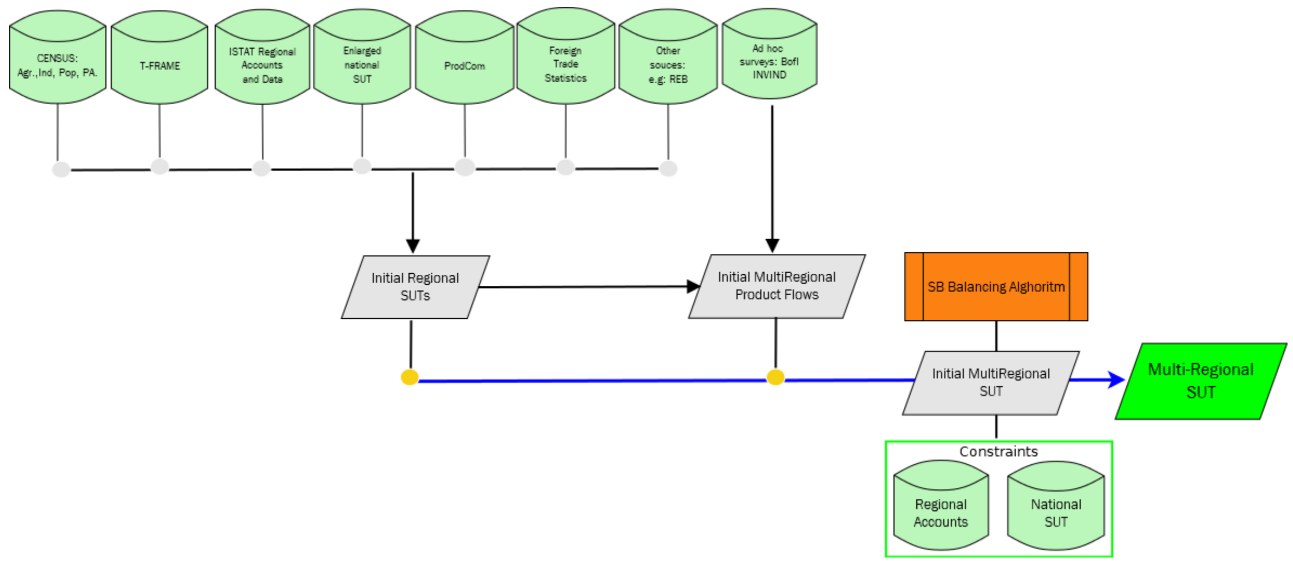
INTRODUCTION

Supply and Use Tables (SUTs) are essential tools for providing a sistemic modelling framework, in order to analyze the economic interdependencies, related to production process, among institutional factors. SUTs support a wide range of analyses, among others: impact assessment, counterfactual evaluation and structural economic analysis. Nonetheless the complexity of the flows recorded in a SUT needs a more intensive effort for estimating them, even more in case of Multi regional SUTs (MRSUTs) which are more data intensive. IRPET has a long standing tradition in the production of this tables, and has spearheaded a number of methodological innovations in this field. Aim of this paper is therefore to describe the most recent IRPET methodology for building a NUTS2 level multiregional Supply and Use Table (henceforth MRSUT), referring to the benchmark year 2019. This working paper should be considered as a methodological (and text) revision and an update of the approach presented in Paniccià and Rosignoli (2018)[1], through a more intensive use of both regional data recently released by ISTAT and *ad hoc* sources of data. In Figure 1 the assembly line of the multiregional SUT.

The structure of the paper is the following: section 1 illustrates the balancing algorithm; section 2 shows how initial estimates have been produced. In section 3 the system of balancing identities is presented along with the main numerical indexes of the balancing algorithm. Three annexes are also provided: i) Annex 1 on the methodology from moving from a MRSUT to MRIO; ii) Annex 2 shows how to transform a MRSUT in a “quasi” Interregional SUT; iii) Annex 3 is about the construction of a multiLMA table for Tuscany.

[1] Paniccià, Renato and Rosignoli, Stefano, "A methodology for building a multiregional Supply and Use Table for Italy", IRPET (2018).

Figure 1.
Multi Regional SUT assembly line



1. THE BALANCING APPROACH

The problem of balancing accounting matrices has been solved in different ways and, before outlining the specific method for solution, it could be helpful to define what is generally meant by "balancing problem".

Let $\Phi(0)$ the initial unbalanced estimate of an accounting matrix of order $r \times c$ and let the vector $\boldsymbol{\tau}(0)$, the vectorization (vec) of this matrix so:

[1] $$\boldsymbol{\tau}(0) = vec\big(\boldsymbol{\Phi}(0)\big)$$

The elements of $\boldsymbol{\tau}(0)$ will be $n = r \cdot c$, indexed by $k=1,n$.

The solution of the balancing problem is to find a new vector $\boldsymbol{\tau}(1)$, from $\boldsymbol{\tau}(0)$, such as:

[2] $$\boldsymbol{\tau}(1) = \boldsymbol{\tau}(0) + \boldsymbol{\varepsilon}$$

that satisfies p set of constraints defined by a set of equations:

[3] $$\mathbf{g}_i \cdot \boldsymbol{\tau}(1) = \mathbf{h}_i \qquad where \quad i = 1, p$$

A balancing problem is therefore a problem of minimization of the distance metric function D[$\boldsymbol{\tau}(0)$,$\boldsymbol{\tau}(1)$] under a set $\mathbf{h}$ of p-nth constraints.

The problem could be difficult for some mathematical and economic issues, particularly:

1. The elements of $\boldsymbol{\tau}(0)$ and $\boldsymbol{\tau}(1)$ are economic flows so their range must be bounded in some way.
2. The size of the vector $\boldsymbol{\tau}(0)$ (and so $\boldsymbol{\tau}(1)$) is generally very high, as the number p of constraints.
3. The structure of $\boldsymbol{\Phi}(0)$ has often a sparse characteristic (and so $\boldsymbol{\tau}(0)$ vector), that generally should remain the same after the balanced solution, this implies implicit constraints to the balancing problem 1.
4. In some cases, the sparsity makes the matrix $\boldsymbol{\Phi}(0)$ not decomposable and so the solution of balancing problem could be unfeasible.
5. Some values of the matrix could be negative (for instance change in inventories or net imports) and for some distance functions (e.g. cross-entropy function) are out of the function domain.

Many solutions have been proposed to solve the balancing problem[2] for our purposes we have chosen and utilized the Stone-Byron algorithm (henceforth SB).

[2] For a review Paniccià and Rosignoli (2018).

The method was firstly proposed by Stone, Champernowne and Meade (1942)[3] (henceforth SCM) to achieve the consistency between national account aggregates, given, a known or inferred, reliability measure of the initials estimates.

Some year after Theil and Schweitzer (1971)[4] shown the statistical proprieties of this method considering its analogy with the GCLS estimator. Following the increasing computing capacity and the use of new computational methods to solve optimization problems, a seminal contribution to the development of the SCM methodology was provided by R.P. Byron (1977 and 1978)[5] the method was utilized to balancing large economic accounting matrices.

The SCM can be seen as the application of a generalized least square estimation subject to constraints of a linear model: this type of estimation is usually solved by way of constrained minimization of Euclidean distance between observed and predicted vector and is known with the name of least squared constrained estimation.
The analogy to the linear model estimation let us to consider as a vector estimate of model parameters, that under the same assumption of the analogous linear model, can be considered best linear unbiased parameter estimators and could be used for it all the statistical testing instruments (e.g. test of significance, test on linear combination of parameter, computation of confidence bound).
The SCM balancing procedure assumes that the initial flows to be balanced are subjected to accounting constraints and can vary according to the relative reliability of preliminary estimate. Instead of the linear bi-proportioning rAs, the concept of variance and covariance, associated to the reliability of the initial accounting set $\boldsymbol{\tau}(0)$ is explicitly introduced.
The solution proposed by the authors consists in a GLS estimator for solving the following problem given an accounting matrix $\boldsymbol{\Phi}$, or vectorization: $\boldsymbol{\tau}(0) = vec\big(\boldsymbol{\Phi}(0)\big)$, subject to a set of constraints **h**, according to the aggregation matrix **G**:

[4] $$\mathbf{h} = \mathbf{G} \cdot \boldsymbol{\tau}(1)$$

Using the initial estimate of $\boldsymbol{\Phi}$(0), , we obtain:

[5] $$\mathbf{h} + \boldsymbol{\varepsilon} = \mathbf{G} \cdot \boldsymbol{\tau}(0)$$

Assuming the initial estimates $\boldsymbol{\tau}(0)$ are unbiased and:

[5.1] $$\begin{cases} \boldsymbol{\tau}(0) = \boldsymbol{\tau}(1) + \boldsymbol{\varepsilon} & a) \\ E(\boldsymbol{\varepsilon}) = 0 & b) \\ E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\Sigma} & c) \end{cases}$$

The use of GLS will lead to the estimate of a vector $\hat{\boldsymbol{\tau}}(1)$ that will satisfy the accounting constraints in [4] and will be as near as possible to the actual data $\tau(1)$. The estimator able to produce such an estimate is the following:

[6] $$\hat{\boldsymbol{\tau}}(1) = [\mathbf{I} - \boldsymbol{\Sigma} \cdot \mathbf{G}' \cdot (\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')^{-1} \cdot \mathbf{G}] \cdot \boldsymbol{\tau}(0) + \boldsymbol{\Sigma} \cdot \mathbf{G}' \cdot (\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')^{-1} \cdot \mathbf{h}$$

It is demonstrated that this kind of estimator is BLU, and its variance is given by:

[7] $$\hat{\boldsymbol{\Sigma}} = \boldsymbol{\Sigma} - \boldsymbol{\Sigma} \cdot \mathbf{G}' \cdot (\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')^{-1} \cdot \mathbf{G} \cdot \boldsymbol{\Sigma}$$

Byron (1978) argued about the analogy of the SCM method with the generalized constrained least square estimation of vector coefficients of a linear model and proposed a version of SCM as pure minimization constrained problem of a quadratic loss function that could be solved through the conjugate gradient algorithm. The SCM estimator can be seen as a solution to a minimization of quadratic loss function of the kind:

[3] Stone, Richard, Champernowne, David G., and Meade, James E., "The precision of national income estimates", *The Review of Economic Studies* 9, 2 (1942), pp. 111-125.
[4] Theil, Henri and Schweitzer, A, "The best quadratic estimator of the residual variance in regression analysis", *Statistica Neerlandica* 15, 1 (1961), pp. 19-23.
[5] Byron, Raymond P., "Efficient estimation and inference in large econometric systems", *Econometrica: Journal of the Econometric Society* (1977), pp. 1499--1515.
Byron, Ray mond P., "The estimation of large social account matrices", *Journal of the Royal Statistical Society. Series A (General)* (1978), pp. 359-367.

[8] $\vartheta = 0.5 \cdot (\hat{\boldsymbol{\tau}}(1) - \boldsymbol{\tau}(1))' \cdot \boldsymbol{\Sigma}^{-1} \cdot (\hat{\boldsymbol{\tau}}(1) - \boldsymbol{\tau}(1)) + \lambda \cdot (\mathbf{G} \cdot \boldsymbol{\tau}(1) - \mathbf{h})$

where λ is the Lagrange multiplier. The first-class conditions for minimizing the previous equation correspond to the following values of Lagrange multipliers:

[9] $\hat{\lambda} = (\mathbf{G} \cdot \boldsymbol{\Lambda} \cdot \mathbf{G}')^{-1} + (\widehat{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} - \boldsymbol{\Lambda} \cdot \mathbf{G}' \cdot (\mathbf{G} \cdot \boldsymbol{\Lambda} \cdot \mathbf{G}')^{-1} \cdot \mathbf{G} \cdot \boldsymbol{\Lambda})$

So that the estimator in [2] will be:

[10] $\hat{\boldsymbol{\tau}}(1) = \boldsymbol{\tau}(0) - \boldsymbol{\Sigma} \cdot \mathbf{G}' \cdot \lambda$

The contribution of R.P. Byron has allowed to overcome one of the main problems that had hindered the use of the SCM procedure in the balancing of significant sets of national accounts and SAM, that is the computational difficulty in inverting the matrix $(\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')$. R.P. Byron proposes the conjugate gradient algorithm to reach an estimate of the Lagrange multipliers, by means of the system of linear equations:

[11] $(\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}') \cdot \lambda = \mathbf{G} \cdot \boldsymbol{\tau}(0) - \mathbf{h}$

Since $(\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')$ is symmetric defined positive, the conjugate gradient method provides a good solution of the coefficients. As also stressed by Nicolardi (2000)[6], even with very powerful computers, this method retains advantages compared to direct estimate using [9] by increasing control provided by the algorithm over possible inconsistencies of the initial estimates $\boldsymbol{\Phi}$ and of $\boldsymbol{\Sigma}$ and by avoiding the numerical instability tied to the inversion of the sparse matrix $(\mathbf{G} \cdot \boldsymbol{\Sigma} \cdot \mathbf{G}')$.

A crucial problem is how to define the variance-covariance matrix that determines, for each flow in $\boldsymbol{\tau}(0)$, the range of adjustment. Both types of minimization problems need to know this matrix. The best procedure would be to estimate each single element of $\boldsymbol{\tau}(0)$ and the variance of its estimator through sampling estimation, but this way is surely very expensive and not so feasible.

The first step concerns the identification of the estimates that are interdependent and or subject to autoregressive processes. This operation is very important because in case of independent estimates the Var-Cov matrix will be diagonal.

Hypothesizing the presence of a diagonal Var-Cov matrix, the next step consists in the estimate of the unreliability of each single data item. The ideal procedure would estimate for each flow the relative reliability, based on its own error profile supplied by the data producers, and therefore would associate it to the matrix of Var-Cov.

Different options could be found in literature, varying from the purely subjective approach to those more and more objective. In the first case an ordinal scale of judgement is formed, that can be associated to ranges of errors (UKCSO).

Byron, Crossman, Hurley and Smith (1995)[7] have followed this procedure and have associated a subjective ordinal scale of reliability to a value of variation coefficient. The subjective reliability judgement is based, principally, on the information of the data producers. A more objective reliability assignment is provided by Van der Ploeg (1982)[8] and by Berker, Van der Ploeg and Weale (1984)[9].

In following articles Weale (1988 and 1993)[10] indicated how it is possible to reach an estimate of the matrix of Var-Cov, without knowing the reliability of the initial data in a dynamic framework, in presence of stationary variance and mean, using as basis the standard deviation over time.

An obligatory step in the determination of the matrix Var-Cov has been the tracing of an assignment paradigm of reliability/unreliability, based on the known economic regional specificities, numerical and constructive characteristics of the initial data.

The model of reliability assignment tries to considerate the factors that could describe the precision of the initial estimate. Once identified, they have been properly combined to determine the unreliability.

[6] Nicolardi, Vittorio, "Balancing large accounting systems: an application to the 1992 Italian IO Table", in *Proceedings of the âœ XIII International Conference on Input-Output Techniquesâ, University of Macerata, Italy, August* (2000), pp. 21-25.

[7] Byron, Raymond P., Crossman, Peter J, Hurley, Jim E, and Smith, SCE, "Balancing hierarchical regional accounting matrices", (1993).

[8] van der Ploeg, Frederick, "Reliability and the adjustment of sequences of large economic accounting matrices", *Journal of the Royal Statistical Society. Series A (General)* (1982), pp. 169-194.

[9] Berker, Terry, Frederick Van Der Ploeg, and Martin Weale. "A balanced system of national accounts for the United Kingdom." *Review of Income and Wealth* 30.4 (1984): 461-485.

[10] Weale, Martin, "The reconciliation of values, volumes and prices in the national accounts", *Journal of the Royal Statistical Society. Series A (Statistics in Society)* (1988), pp. 211-221.

Solomou, Solomos and Weale, Martin, "Balanced estimates of national accounts when measurement errors are autocorrelated: the UK, 1920-38", *Journal of the Royal Statistical Society. Series A (Statistics in Society)* (1993), pp. 89-105.

The guidelines of the reliabilities/unreliabilities assignment have therefore led to a mixed subjective-objective[11] technique.
Most applications (see for instance Stone 1990)[12] show that such unreliability is transformed in variance through equation [12], that is:

[12] $$\sigma^2_{r,i,j} = \left(r_{r,i,j} \cdot \tau(0)_{r,i,j}\right)^2$$

It is important to notice that variance in relative terms affects the balancing process, so if the matrix **V** is multiplied by a scalar, there is no modification of the result.
In the literature, the matrix **V** has nearly always developed a diagonal form, this implies initial estimates derived from independent sources. The condition of non-diagonality can be released when it is supposed that the preliminary estimates are not independent. We can also release the diagonal condition when implicit covariance in the production of the in initial estimates is proved. This happens when:
a) the same initial estimate figures in more than one account.
b) the estimate of a flow implies using data in other account.

Many authors argue that also autoregressive processes should also be considered, even in uni-temporal applications, within the Var-Cov matrix (Antonello 1990)[13].
However, the main reasons why SB has been preferred to other methods has well been summarized by Round (1990)[14]. Presenting a review of the balancing methods applied to Social Accounting Matrices (rAs, Cross Entropy and SCM) the author clearly expresses his opinion in conclusion (p. 179, par.3): "… *In spite of the apparent preference for the Cross Entropy method by many compilers of SAMs, the Stone Byron method (possibly extended to include additional constraints) does seem to have some advantages over alternative methods. It allows us to incorporate judgement on the relative reliability of data sources and it is therefore closer to the spirit of the problem at hand*".

2. INITIAL ESTIMATES

In this chapter[15] will focus our attention on a crucial aspect of balancing procedure, that is the initial estimates and on six components of MRSUT that is*:* *i*) value added and output, *ii*) use intermediate matrix, *iii*) make matrix, *iv*) domestic final demand, *v*) foreign import and export, *vi*) interregional trade.
Before starting, just to remind (see Figure 1) that an enlarged national SUT table (henceforth eSUT) has preliminary been estimated through: *i*) increasing the number of sectors (see Appendix 2): *ii*) disaggregating the household expenditure not only by consumption products but also by consumption function COICOP-12, in order to define a bridge consumption matrix cpa64-COICOP-12: *iii*) for the public administration expenditure a bridge matrix cpa63-COFOG-10 has been estimated; iii) regarding gross fixed investments, two bridge matrices have been computed: the first one links investments products (cpa63) to investments by assets (P51), the second one investments by owner sector (rr28) to investments by asset (P51).

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[11] Indeed, for some variables we could utilize the sampling standard deviation as provided by ISTAT. This is the case of the total intermediary costs drawn by TFRAME.
[12] Stone, Richard, "Adjusting the national accounts", Lecture presented at the Central Institute of Statistics, Rome, September 1988. In Nuova Contabilità Nazionale, Annali di Statistica, Ser. IX, vol. 9, ISTAT, Roma.
[13] Antonello, Paola. "Simultaneous Balancing of Input-Output Tables at Current and Constant Prices with First Order Vector Autocorrelated Errors." *Economic Systems Research* 2.2 (1990): 157-172.
[14] Round, Jeffrey, "Constructing SAMs for development policy analysis: lessons learned and challenges ahead", *Economic Systems Research* 15, 2 (2003), pp. 161–183.
[15] Just a quick remind of the symbology used henceforth:
a) rrXY = sector type of aggregation, for instance: rr43 means 43 sectors Nace Rev. 2.
b) rrXY:xy = sector xy of the Nace Rev.2 classification included the rr43 aggregation, for instance rr43:B is "Fishing"
c) cpaXY= CPA product aggregation, for instance: cpa54 means 54 CPA 2.1 products.
d) cpaXY:xy = product xy of the CPA 2.1 classification included the cpaXY aggregation, for instance cpa63:10-12 is "Food products, beverages and tobacco products"
e) ABCD-XY= ESA2010 nomenclature type of aggregation, for instance: COICOP-12 means the twelve COICOP consumption function
f) RegAcc = official regional accounts released by ISTAT
f) $\tilde{x}$ = initial estimate of variable x
g) $\mathbf{x}$ = vector
h) $\mathbf{X}$ = matrix

2.1 USE MATRIX

2.1.1 Value ADDED and PRODUCTION

Value added at basic prices is regionally released by ISTAT, as part of the Regional Accounts (henceforth RegAcc), at 28 sectors (henceforth rr28), in a more aggregated way of what we need (43 sectors, henceforth rr43)[16]. Except for agriculture, financial services and PA services, the initial estimates of missed rr43 sectors are based on the Territorial-FRAME-SBS data (henceforth TFRAME), where value added, at factors cost (**vafc**), are available at LKAU level. For the remaining sectors (except for agriculture) the aggregated rr28 regional value added has been broken down using the share of employment drawn by CENSUS for the benchmark year (2019). The rr43 initial estimate of regional valued added at basic prices is the result of a 3DGrAs (Eq. 13) for making it consistent with the following constraints at basic prices: i) RegAcc rr28; ii) the national rr43 value added at basic prices.

[13] $$\widetilde{\mathbf{vapb}} = 3DGrAs\left(\mathbf{vapb}_{rr28}^{RegAcc}, \mathbf{vapb}_{rr43}^{eSUT}, \widetilde{\mathbf{vafc}}\right)$$

Regarding production, the TFRAME does not supply, at regional and LKAU level, the correspondent value but only the amount of turnover. For getting a more proxied initial estimate of output we sum up value added at factor prices with the total cost of intermediaries, both available at LKAU level. This proxy variable od production will be also used for estimating sectoral industry-mix (see later section 2.1.2).

The initial estimate of production at basic prices ($\widetilde{\mathbf{xbp}}$) has been computed by spreading across the regions the sectoral output from the national SUT according to the regional shares of production proxy from T-FRAME. This represents the general approach in computing initial production, indeed for some sectors it is possible to use additional data source and trying to refine and strengthen the initial estimate. The most significant case is represented by the primary macro-sector (agriculture, forestry, and fishing). Indeed, ISTAT is providing not only, at regional level, for agricultural sectors: output, value added and intermediate costs at basic price, but also data on the agricultural, fishery and forestry products at cpa54 level.

2.1.2 INTERMEDIATE FLOWS

In the benchmark year (2019) the estimate is characterized by two sequential steps. First, the initial $\mathbf{B}$ regional coefficients matrices are obtained by means the industry-mix approach[17] (hereafter IMIX). This implies the regionalization of the cpa54:rr63e national **B** matrix, into the rr43 regional industries. This procedure allows us to catch the regional diversities tied to the sectorial specialization in the composition of each single regional rr43 sector input structure. The aggregation by means IMIX has come about according to the following equation:

[14] $$\tilde{b}_{rij} = \sum_{k=1}^{ns(j)} b_{ik}^{eSUT} \cdot QD_{rik} \qquad \forall\ k\ \in\ j$$

where:
ns(j) = number of the rr63e industry which belongs to j-nth rr43 sector.
QD= industry-mix of j-nth rr43 sector of the r-nth region based on the TFRAME production proxy.

Once available the regionalized $\mathbf{B}$ coefficients, this allows to compute the initial Use matrices using the initial estimate of output at basic prices (see previous paragraph), so:

[15] $$\tilde{u}_{rij} = \tilde{b}_{rij} \cdot \widetilde{xpb}_{rj}$$

This type of regionalization it is not sufficient to encompass regional peculiarities linked to, for example, specific regional technologies. Hereafter we will discuss some particular cases for which we have information on a regional and sectoral basis which allow us to introduce specific regional $\tilde{b}_{ik}$.

[17] Shen, T. Y. "An Input-Output Table With Regional Weights." *Papers in Regional Science* 6.1 (1960): 113-119.

2.1.3 SPECIAL SECTORS

- Agriculture, Forestry and Fishing (rr43:AA and rr43:AB)

ISTAT provides detailed regional accounts on agriculture, forestry and fishing sectors, not only in output quantity but also in terms of monetary production, value added, and some particular intermediate costs (ie, energy expenditure).

- Coke and refined petroleum products (rr43:CD)

From the regional energy balances (REB) provided by ENEA in the annual RAEE, it is possible to estimate the regional share of crude oil entering as input cost in the r43:CD sector. This is important not only for the information per se but also for correcting the input cost structure from the effects of multiplant enterprises with different specialization. The higher is the incidence of headquarters on the regional output of sector rr43:19 the closer to a business service sector will be the input cost structure, on the contrary the highest is the share of crud oil entering as input the closest the input cost will be linked to the refinery and coke production.

- Electric power generation (NACE-rev.2:35.11)

Electric power generation is a sub-sector of rr43 industry "D: Electricity, Gas, Steam and Air-Conditioning". At regional level we do have some significant information on energy products utilized for generating electric power from REB. The higher is the percentage of renewable used for producing electricity the lower is the input needed from two cpa54 products that is: B Mining and quarrying (natural gas and coal), and cpa54:19 Refinery and coke (Fuel Oil). Not only, amongst the renewables we also have information about how much of them are from biomass (cpa54:2 Forestry and cpa54:16 Wood and wood products) and from non-renewable wastes (cpa54:37 Waste management). Based on this information a specific input cost structure for electric power generation has been assigned to each region and introduced in the equation [15].

2.1.4 DOMESTIC FINAL DEMAND

In the base year, in the regional SUT household expenditure expressed by a bridge matrix connecting the COICOP-12 consumption functions with the cpa54 consumption products. The ISTAT RegAcc provides the regional COICOP-12 values, so the initial estimates has been made by multiplying these values by the shares of cpa54 products by COICOP-12 derived from the national eSUT. Same procedure for the cpa54-COFOG-10 matrices of regional SUT, in this case ISTAT RegAcc supplies the regional PA expenditure by COFOG-10.

Information on Gross Fixed Investments provided by RegAcc are: Investments by demanding sector at rr28 and Investments in Construction so the calculation of the initial cpa54-P51 gross fixed investment bridge matrix for any region has been computed through the following steps: i) investments at rr28 have been transformed in investment by assets through the national corresponding bridge matrix, ii) then the regional vectors of the resulting investments by asset have been distributed through the cpa54 investment products using the corresponding bridge matrix from the national eSUT. Paradoxically with the availability of T-FRAME-SBS data the initial estimate of Changes in inventories by sector producer has resulted easier and more reliable because could be proxied by the difference between output and turnover as expressed in T-FRAME-SBS data. Data by sector has then been transformed in products by using the make matrix initial estimates.

2.1.5 FOREIGN EXPORT

Foreign exports of goods are released by ISTAT, region-wide, at 3 digits NACE-rev.2 and they are expressed at fob and purchasing prices. The first adjustment to be operated is the transition of export flows from fob to cif and then the extraction of taxes on products and trade and transport margins. Both operations should be done using national parameters, for taxes, trade and transport margins we have utilized the corresponding matrices available at national level, so for each i-nth cpa54 products and r-nth region, the initial values of foreign export of goods at fob and basic prices is computed as following:

[16] $\widetilde{ew}_{ri} = ew_{ri}^{cif,pp} \cdot \kappa_i \cdot (1-\tau_i) \cdot (1-\vartheta_i)$

where:
κ = ratio fob/cif value at national level for the i-nth product
τ = taxes on products ratio for the national foreign export of the i-nth product
ϑ = trade and transport margins coefficient for the national foreign export of the i-nth product

Once computed the initial ew_{ri} values have been re-balanced with the corresponding national SUT values.
Trade and transport margins extracted through [26] have then been added to the initial estimates of the corresponding cpa54 products, as taxes on products will be assigned to foreign export taxes.
Regarding export of services, they are available at regional level from ISTAT-ICE report as export credits, as for goods, for making the initial estimates consistent we have extracted taxes on products as in [26] and then rebalanced to foreign export values from national SUT.

2.2 SUPPLY MATRIX

2.2.1 MAKE MATRIX

The make matrix explicit the dichotomy between sectors and products in the production process and it is the key matrix for determining technology and aggregation in computing symmetric I-O tables.

Unlike the Use matrix there are no information available on the output of products at regional level because the ProdCom survey on manufactured goods is representative at national level, but we could proceed, for the benchmark year, through refinement steps, trying to use all information available from other sources.
The first steps, is computing, starting from national product-mix matrix $\mathbf{C}$ coefficents at rr63e, the regional initial estimates of $\mathbf{C}$ trough IMIX:

[17] $\tilde{c}_{rji} = \sum_{k=1}^{ns(j)} c_{ki}^{NATeSUT} \cdot QD_{rki} \qquad \forall\ k\ \in\ j$

where:
QD = industry mix operator as in [14].
Once estimated the regional **C** matrices we could obtain the regional Supply matrices through output at basic prices:

[18] $\tilde{s}_{rji} = \tilde{c}_{rji} \cdot \widetilde{xpb}_{rj}$

As for the Use matrices it is possible, after IMIX, to refine the initial estimates of subsets of the regional supply matrices by using other data sources. We could divide this refinement in two types: i) in the first case the adjustment has been made before the aggregation process by introducing specific regional c_{ik}; ii) in the second type, the initial estimate of supply table cell is obtained by a breakdown of national supply table value through a regional share indicator.

- Farm holidays output (NACE-rev.2:55.20.52)

Indeed, the product output of primary sector (agriculture, livestock forestry and fishery) is supplied by the ISTAT a very detailed level region wide not only for the typical product but also for the related and support services. Nonetheless especially in some case farms are also producers of accommodation and food services (farm holidays). This type of tourism is particularly relevant in some regions like Tuscany. This implies that supply matrix cell crossing sector rr43:AA and cpa54:I. In producing the initial estimate we have utilized the second type of adjustment so the each single regional supply cell crossing rr43:AA and cpa54:I has been estimated through the breakdown of the national supply table value according the incident of night spent in farm holidays:

[19] $\tilde{s}_{r;rr43:AA,cpa54:I} = s_{rr43:AA,cpa54:I} \cdot \frac{nh_r}{nh}$

where:
nh = night spent in farm holidays for region r-nth

- Oil and Gas extraction (r63a:06)

The regional information about the output product of crude oil and natural gas from Minister of Economic Development (MISE) enters in the industry mix aggregation procedure of C matrix by introducing a regional specific rr63a c_{ik} value crossing: rr63a:Oil&gas extraction, cpa54:Mining and quarrying. In this case the importance relies in the fact that some regions are very specialized in extracting fossil fuels like Basilicata as others are less devoted to extraction and more to providing services because headquarter of multiplant enterprises.

- Coke and refined petroleum products (cpa54:19)

Refinery and Coke output product crosses supply table at rr43:CD and cpa54:19 and the regional specific value has been computed through a second type of adjustment that is:

[20] $$\tilde{s}_{r;rr43:CD,cpa54:19} = s_{rr37:CD,cpa54:19} \cdot \frac{Ref_r}{Ref}$$

where:
Ref = Refineries output for region nth expressed in ktoe from REBs

- Electric Power generation (NACE-rev.2:35.11)

Electric Power generation output product crosses supply table at rr43:EP and cpa54:EL The regional specific value has been computed using as spreading indicator the regional MW produced:

[21] $$\tilde{s}_{r,rr43(21),cpa54(13)} = s_{rr43(10),cpa54(23)} \cdot \frac{mw_r}{mw}$$

where:
mw = megawatt produced in the region *r-nth*

- R&D production (cpa54:72)

R&D output is a typical secondary product of many manufacturing enterprises (*intramuros* R&D) besides the specific output of sector rr43:MB. ISTAT publishes the whole production both *intramuros* and *extramuros* R&D at regional level. The *extramuros* crosses regional supply tables at rr43:MB, cpa54:72 as the breakdown by sectors rr43 of the total *Intramuros* has been computed through the national intensity of *intramuros* R&D over total output as:

[22] $$\widetilde{rd}_{rj} = rd_r \cdot \frac{rd_j}{xpb_j}$$

2.2.2 FOREIGN IMPORT

Firstly, foreign import has been broken down in two groups: intermediate and final import. This distinction is extremely important in terms of economic analysis and implies an additional effort of estimate. As for export, imports of goods at three digits NACE-rev.2 are supplied by ISTAT through the CoeWeb dataset as for services using the credits/debits in the Bank of Italy database.

In the following part the general procedure just for manufacturing goods is presented.
First, the 3 digits NACE-rev.2 regional imports at cif prices are aggregated at cpa54 and in two groups: intermediate and final destination. This operation has been possible by using the MIG classification[18] which allow to distinguish 3 digits products by destination that is: energy, intermediate, final durables, final non-durables, final investments, and services. Energy and intermediate will constitute the cpa54

[18] The Main Industrial Groupings, abbreviated as MIG, provide an alternative statistical breakdown of the economic activities of Manufacturing, as compared to the sectoral breakdown of the Statistical classification of economic activities in the European Union (NACE).
The MIG is at an intermediate level between the NACE Sections on the one hand and the Divisions and Groups on the other.
There are five MIG: intermediate goods; capital goods; consumer durables; consumer non-durables; energy.

intermediate import.

Second, once defined two cpa54 vectors for each region, that is: intermediate ($\widetilde{\mathbf{mwi}}^*$) and final ($\widetilde{\mathbf{mwf}}^*$) foreign import, the $\mathbf{MWI}$ matrices have then been re-scaled by comparing their sum with the national $\mathbf{MWI}$ drawn by the national SUT, this will also allow to correct the import flows of products for cif/fob evaluation. The new initial estimate of $\mathbf{MWI}$ for the r-nth region will be:

[23] $$\widetilde{\mathbf{mwi}}_r = \widetilde{\mathbf{mwi}}_r^* \cdot \frac{\mathbf{mwi}}{\sum_{r=1}^{nr} \widetilde{\mathbf{mwi}}_r}$$

Same procedure has been developed for foreign final import. As for the intermediate import, $\mathbf{MWF}$ matrices have then been re-scaled to the national $\mathbf{MWF}$, drawn by the national SUT, so the new initial estimate of $\mathbf{MWF}$ for the r-nth region will be:

[24] $$\widetilde{\mathbf{mwf}}_r = \widetilde{\mathbf{mwf}}_r^* \cdot \frac{\mathbf{mwf}}{\sum_{r=1}^{nr} \widetilde{\mathbf{mwf}}_r}$$

As the Handbook on Supply and Use Tables and Input Output-Tables (United Nations 2018)[19] recommends, foreign imports should be assigned to the final destination region that is the actual demanding region, nevertheless there are problems of identifying ultimate regional destination by using the regional CoeWeb import data for some products. Because of the way they are recorded[20] it could be found that foreign imports of some products are not significantly tied to the regional demand. Two examples amongst others. First, according to import data Tuscany should be one of the most important regions demanding foreign cars. Indeed, Tuscany is an important access point through the Leghorn harbor where imported cars are stored before to be delivered to other regions, the same for Veneto, in this case the crucial role is played by the Verona inland terminal. Second example is the import of natural gas trough pipelines or regassification plants. In the first case is not assigned to a specific region, in the second case import of gas is assigned to the region where the plant is located.

After a careful examination of import and demand data we came to the identification of such as problem for some cpa54 products[21]. Once defined them we replaced the $\mathbf{MWI}$ and $\mathbf{MWF}$ corresponding rows by spreading the corresponding national values according to the regional intermediate/final demand.

2.3 MULTIREGIONAL TRADE

The trade flows of goods and services between couples of regions is a crucial aspect in the construction of I-O regional tables, since it is through these bilateral trade flows that the most appropriate matrix of multiregional transactions per product can be derived. Despite the importance of this phenomenon, the information sources available in Italy are relatively scarce. Particularly relevant for the purposes of our analysis are:

A) the sample yearly survey Road Freight Transport (RFT) by Istat, which records the flows of quantities of goods expressed in tons transported by road from one region to another, broken down into commodity macro-sectors.

B) the sample surveys of the Bank of Italy (INVIND)[22] on manufacturing and service KAUs, which, for 2009, recorded the turnover "exported" from the NUTS2 region where the KAU is located to the geographical NUTS1 macro area of destination.

The main pro of RFT is that it details trade from region to region, rather than from region to geographical area like INVIND. On the other hand, INVIND has some advantages vs RFT:

1. includes trade flows related to the services sector, which are not registered in RFT;
2. INVIND is using the ATECO classification, unlike RFT adopts the commodities transport (NST/R);

[19] Handbook on Supply and Use Tables and Input Output-Tables with Extentions and Applications 2018, New York: United Nations.

[20] In many cases import is assigned to enterprise and so the region of the headquarter instead the LKAU using the imported product ii) In other cases to the access point region (see for instance inland terminal, harbor, airports, regassification plants for natural gas).

[21] cpa54:05, cpa54:11….

[22] This is a unique and exclusive database, which for 2009 for the first time collected, inter alia, information on the turnover of 1,706 industrial firms and 697 service firms with 50 or more workers; most of these firms (1,338 and 624 respectively) also provided details of the breakdown of turnover between the four geographical macro areas, which were then used for the estimates.

3. RFT records tons of freight and the trade size of each sector is clearly affected by the commodity related composition.

Giving *pros* and *cons* of both sources we decided to use the Bank of Italy survey has key dataset for estimating the product multiregional intermediate[23] flows amongst regions through a deterrence function included in (LS) formulation of multiregional trade.

The LS algorithm has the following specification as:

[25] $$t_{rs;j} = \frac{z_{r;j} \cdot z_{s;j}}{z_j} \cdot \delta_{rs;j}$$

where:

$t_{rs;j}$ = intermediate flow delivered from sector j-nth of region r-nth to region s-nth,
z_{rj} = total intermediate output, net of foreign intermediate exports, of sector j-nth belonging to region r-nth (supply pool),
z_{sj} =intermediate domestic demand, net of intermediate imports from abroad of sector j-nth of the destination region s-nth (demand pool),
z_j = scale factor, total intermediate production net of sector j foreign intermediate exports,
$_{rsj}$= deterrence factor.

The estimate of the deterrence factor proves to be crucial to the estimate of the gravity model in [13]. In the case of trade flows this parameter represents the transaction costs between the two areas, without which the origin and destination flows would be simply driven by the concentration of supply and demand. To isolating the effect of the "transaction costs" and, in turn computing the deterrence factor, we could approximate it through:

[26] $$\tilde{\delta}_{rs;.j} = \frac{t_{rs;.j}}{t^*_{rs;j.}}$$

where:

$t^*_{rs;j}$= represents the value of the theoretical flow of goods/services that there would be without the "transaction costs" between the two areas.

The variable $\delta_{rs;j}$ illustrates the impact of such costs on bilateral commercial trade: if it is less than 1 the "transaction costs" depress the volume of trade; if it is greater than 1 these "costs" are fairly low, and the trade thus proves to be particularly intensive. The variable $\delta_{rs;.j}$ can therefore be used as a dependent one in a model (deterrence/decay function) that includes among the regressors all the factors that influence the "transaction costs" and, through these, the trade flows between geographical areas so:

[27] $$\delta_{rs;.j} = f(x(1)_{rs}, .., x(i)_{rs}, ..., x(l)_{rs}) \qquad i = 1, l$$

The LS formulation will become:

[28] $$t_{rs;j} = \frac{z_{r;j} \cdot z_{s;j}}{z_j} \cdot f(x(1)_{rs}, .., x(i)_{rs}, ..., x(l)_{rs})$$

2.3.1 MULTIREGIONAL INTERMEDIATE FLOWS

The steps for computing initial estimates of intermediate multiregional flows from INVIND dataset have been the following:

1. Specification and estimate of the deterrence function at NUTS1 level,
2. Estimate of interregional NUTS2-NUTS2 flows by using the corresponding NUTS1 deterrence functions parameters,
3. because survey data are by sectors, we have transformed the rr43 sectors interregional flows to cpa54 flows using the corresponding initial estimates of the regional Make matrices.

[23] This is the main upgrade from the procedure adopted in where INVIND was utilized as dataset for the estimate of the undistinguished final/intermediate interregional product flows.

Following the literature, the main explanatory variable in the deterrence function is the distance, which is generally proportionate to the expense of trading the commodities. Distance, in turn, is greater or lesser depending on the extension of the network of links and the provision of infrastructures within the territory; for this reason, reference is made to effective distance[24]. The importance of the two areas, in economic terms, ought to influence the reciprocal trade. We would expect that the greater the economic weight of the area, the more significant the flows of products and services sold elsewhere will be; moreover the level of economic development within the area could render the demand for goods and services originating from other areas less significant so the higher is the distance in term of economic development the lower should be the propensity to trade intermediate goods For the purposes of this analysis the economic impact of an area is considered approximate to the per capita GDP. Another variable to be borne in mind in the analysis of the trade of intermediate goods and services between geographical areas is related to multiplant enterprise which localize LKAU in different regions. In this case, part of the flow of goods between the two areas is not determined by demand and supply but rather by the intra-industrial trade, so that it does not precisely reflect the sectoral interdependencies between the areas (). To take this phenomenon into consideration, INVIND have recorded the distribution by geographical macro area of the employees of all KAUs in the sample with at least 50 workers. The assumption is that the greater the number of workers of an enterprise localized in another area, the more intensive and frequent the trade of intermediate products. The commodity type of the products traded also determines greater or lesser "transaction costs": for example, transporting slabs of marble costs more than transporting toys. Here we are talking about the so-called "tradability effect", and to take this into account it we have introduced a sectoral dummy variable. The regression model used, which sets the "transaction costs" between two NUTS1 regions (approximated by the variable\delta defined above) in relation to the principal factors influencing trade between them, has a log-log functional form and it can be expressed as follows (expected signs):

[29] $$\ln(\delta_{RS;j}) = \beta_0 + \beta_1 \cdot ln[(ED_{RS})^{-1}] - \beta_2 \cdot ln\left(\frac{GDPpc_R}{GDPpc_S}\right) + \beta_3 \cdot ln(NE_{RS;j}) + \beta(j)_4 \cdot \ln(SED_{RSj})$$

where:
ED_{RS} = effective distance (or closeness) between the NUTS1 area of origin R-nth and destination S-nth, measured through API Google application.
GDPpc = average per capita GDPs (1995-2006) of the origin and destination macro areas. The ratio of the two GDPs provides a relative measure of the economic closeness of the two areas.
NEj = average number of employees (1995-2006) belonging to KAU in NUTS1 r$-nth$ that are permanently employed in production LKAUs located in NUTS1 s-*nth*
SED_{RSj} = interaction variable between j sector economic activity (type of goods produced) and the distance between macro areas R and S, obtained as a product of the variable ED and a sector dummy, which ought to (at least partially) take in the tradability effect.

The estimates were performed separately for the manufacturing and the services sectors because tradability and transaction costs associated with the trade of commodities can be very different from those of services The estimates were performed using OLS and are robust for heteroscedasticity and for the clustering effect for pairs of areas. The results of the estimates are illustrated in Table 1.

Table 1.
RESULTS OF THE ESTIMATES OF THE DETERRENCE FUNCTION, DEPENDENT VARIABLE: ACTUAL/THEORETICAL TRADE RATIO

Regressors	Manufacturing	Services
$\log(ED^{-1})$	0.268437	0.600327
$\log\left(\frac{\text{GDPpc}_r}{\text{GDPpc}_s}\right)$	-0.090194	0.415392
$\log(NE_{rs})$	0.11505	
constant	-0.25412	-0.350288
N	157	48
R^2	0.294	0.281

Source: Cherubini and Paniccià (2012)

[24] The effective distance between two areas has been proxied by the average crossed travel time between the NUTS3 components of the two areas computed through Google API.

In general, we should note that, in comparison to estimates made in other studies, the availability of information on the destination of the turnover and the breakdown of employees by geographical area appears to weaken the distance effect which, although it remains the most important factor, nevertheless reveals a more modest impact[25]. Moving on to the original multiplication model, we would obtain the predicted values of the deterrence function for any NUTS2 region level by extrapolating the relationships computed at NUTS1 level, so that:

$$[30] \qquad \tilde{\delta}_{rs;j} = \beta_0 \cdot [(ED_{rs})^{-1}]^{\beta_1} \cdot \left(\frac{GDPpc_r}{GDPpc_s}\right)^{\beta_2} \cdot \left(NE_{rs;j}\right)^{\beta_3} \cdot \left(SED_{rs;j}\right)^{\beta(j)_4} \qquad \nabla r \in R \qquad \nabla s \in S$$

Then we could specify the LS relationships for any j-nth sectoral intermediate multiregional, from origin region r-nth and to destination region s-nth:

$$[31] \qquad \widetilde{tx}_{rs;.j} = \frac{\tilde{z}_{rj} \cdot \tilde{z}_{sj}}{z_j} \cdot \tilde{\delta}_{rs;.j}$$

where:
z$_{sj}$ = initial estimate of regional intermediate demand of sector j-*nth*, net of intermediate foreign import estimated by using the initial regional s-nth regional Use matrix and market shares from initial Make matrix.
So:

$$[32] \qquad \tilde{z}_{sj} = \sum_j \sum_i \tilde{d}_{ji;s} \cdot \tilde{u}_{ij;s}$$

z$_{rj}$ = initial estimate of regional intermediate demand of sector j-nth, net of intermedite foreign export.

Indeed this last aggregate is not estimated through the initial Use and Supply so we need to computed it $ex - novo$. by using the initial regional s-nth regional Use matrix and market shares from initial make matrix, so by crossing the MIGs and ATECO2007 classification we could compute the number of employers embedded in the intermediate production by rr43 sectors (Ex), and using this information for spreading the intermediate output, derived from national SUT, over the regions according to the following formulation[26]:

$$[33] \qquad \widetilde{xpbx}_{rj} = xpbx_j^{natSUT} \cdot \frac{1}{1 + \frac{Ex_r}{Ex} \cdot \frac{\frac{1}{\frac{Ex_{rj}}{Ex_j}} - 1}{\frac{Ex_{rj}}{Ex_j}}}$$

so:

$$[34] \qquad \tilde{z}_{rj} = \widetilde{xpbx}_{rj} - ewi_{rj}$$

Nonetheless the multiregional trade in the initial SUTs is expressed by product so we should move from sectors to products by using the initial estimate of regional make matrices, particularly the product shares. The initial estimate of intermediate trade between region r-nth and the region s-nth for the i-nth cpa54 products is defined as:

$$[35] \qquad \widetilde{tx}_{rs;i} = \sum_{j=1}^{ns} d_{rij} \cdot \widetilde{tx}_{rs;j} d_{s;ji}$$

where:
d$_{rji}$ = market share of origin region product i-nth
d$_{sji}$ = market share of destination region product i-nth (transpose)
ns = number of rr43 sectors

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[25] For a deeper discussion on the estimate results see: Cherubini, Luca and Paniccià, Renato, "A multiregional structural analysis of Italian regions", *Macroeconomic modelling for policy evaluation* 120 (2013), pp. 101--133.

[26] By construction this formulation "rewards" region and sector with a LQ greater than 1 This mathematical formulation has been adapted to foreign export and it has been drawn from: Costa P. and Martellato D. (1987), Intereg: un modello multisettoriale multiregionale per l'Italia in Ricerche quantitative e basi statistiche per la politica economica, Roma, Banca d'Italia: 1–47

2.3.2 MULTIREGIONAL FINAL DEMAND FLOWS

For domestic final demand we intend the sum, by product, of five final demand components, that is: household expenditure, PA expenditure, NPIHS, Gross fixed Investment and Acquisition less disposal of valuables (AIDV). We suppose that interregional flows of final demand is led only by the concentration pool of supply and demand, which means, in terms of LS formulation, a unity value of factor δ. As for the intermediate products, agglomeration and production chains are very significant in explaining origin and destination of interregional flows, in determining the final demand import/export trade sector is becoming more and more important the large-scale retail channels in influencing the destination of final production. In a relative "not big" country like Italy, distances (transportation costs) tend to be overwhelming by concentration of final demand. Just a evidence, from the national SUT it is possible note that trade and transport margins accounts for 11% of intermediate goods cost the percentage rise up to 40% for consumption goods and 30% for investments goods. The initial estimates for final demand flows of product i-nth from origin region r-*nth* to destination s-*nth* will be defined as:

[36] $$\widetilde{tf}_{rs;.i} = \frac{\tilde{f}_{r.;.i} \cdot \tilde{f}_{.s;i.}}{f_{..;i}}$$

where:
f_{si} = initial estimate of regional domestic final demand of product i-nth, net of final foreign final import
f_{ri} = initial estimate of regional domestic final output of product i-nth, net of final foreign final export

2.3.3 MULTIREGIONAL SPECIAL PRODUCTS FLOWS

Not all trade flows products (final and intermediate) have been estimated by using the procedures presented above. In some cases, we have derived the initial estimates in different ways because of i) institutional causes; *ii*) typical features of the products; *iii*) availability of more information on multiregional trade flows.

- Mining and quarrying (cpa54:B), natural gas and crude oil

The flows of cpa54_B is mainly made up by two components: extraction of coal, oil and natural gas and other minerals so the trade flows of the product table is the sum of these two parts. As for the other minerals the procedure in 2.6.1 is used for the energy mining we have used information from MISE and Unione Petrolifera for determining the trade flows in quantity from producing regions, mainly Extra-region (offshore oil and gas fields) and Basilicata (on shore).

- Public Administration and defense services (cpa54:84)

In a multi-regional trading system, there are flows that are not attributable to endogenous processes in the economic system and/or KAUs multi-localization. We refer to non-market services related collective consumption (CCS)[27] trade flows of such as type of services is due to the highest density of local units of national and local public institutions in certain areas/regions of the country. It would seem counterintuitive that CCS could be exported or imported, but when comparing, regionwide, the regional output of these services with the domestic expenditure, it is noticed that most regions show a production deficit, while only a few records a significant surplus. Since there is no foreign trade, such deficits / surpluses can only be balanced by assigning them to interregional import/export. The presence of unbalanced CCS accounts arises because demand for them is recorded in per capita basis as output according to the output of LKAUs. Indeed, it is no coincidence that the region that shows the strongest surplus is the capital region Lazio where the localization of central government headquarters produces much more than the demand expressed on a per capita basis from the inhabitants of that region[28]

[27] According to the ESA2010, the PAs production responds to two different types of demand: 1) from the household sector for individual consumption; 2) from the whole community (collective consumption). In the COFOG classification, the following PA expenditure functions can be defined as collective consumption: 1. General services; 2. Defense; 3. Public order; 4. Economic affairs; 5. Environmental protection; 6. Housing and community protection; all of them are in the cpa54:88 product.

[28] About the importance of such as flows, the zeroing of interregional balance of product cpa54:84 would cause for the capital region a change in total net import, which would be positive.

Given that, the initial multiregional trade flows have been built through the following procedure: i) identification of net exporter/ importer regions; ii) spreading of the multiregional surplus from the net exporter regions overt the net importer regions on a per capita basis. Hereafter the formulation:

if: $xpb_{r;cpa54:84} > cc_r$ (net exporter)

[37] $$\tilde{t}_{rs;cpa54:84} = \widetilde{xpb}_{r;cpa54:84} \cdot \frac{pop_s}{pop} \qquad \forall r, s$$

if: $xpb_{r;cpa54:84} < cc_r$(net importer)

[38] $$\begin{cases} \tilde{t}_{rr;cpa54:84} = \widetilde{xpb}_{r;cpa54:84} & (a) \\ \tilde{t}_{sr;cpa54:84} = \widetilde{xpb}_{s;cpa54:84} \cdot \frac{pop_r}{pop} \qquad \forall r \neq s & (b) \end{cases}$$

where:
cc = collective services demand
pop = territorial population

- Construction (cpa54:F) and Repair and installation services of machinery and equipment (cpa54:33)

In compiling accounts ESA2010 suggests the resident approach, nonetheless activities of the Construction and repairing would be allocated to the region where the building site or machinery to repaired are located no matter the residence of who is doing building or repairing. In turn this means that external flows are almost null apart from very cases like ordinary maintenance for construction, and the trade flows matrices of both products are almost diagonal.

- Real estate services (cpa54:L), imputed rent

Three are the main components of real estate services: i) imputed rent ii) effective rent iii) other housing services. The first two housing services are the most important components of this product. For effective rent and first owned dwelling the territorial approach identifies the dwellings are LKAUs of the host region producing effective rent almost excluding any type of trade flows. In case a household is owning a second dwelling in another region used by the owning household for own final consumption, the rental value should be registered as an interregional export from the region where the dwelling is located to the region where the owner resides. The latter region thus imports this service and uses it for final consumption expenditure of households. As in the case of mixed income, the operating surplus resulting from this production process will differ from the operating surplus in the allocation of primary income accounts of households. So for cpa54:L the most important trade flows is determined by the imputed rent of second dwellings of non-resident owners. The counterpart of the imputed rent in the households account is the household gross operating surplus, and from the ISTAT regional household accounts it possible to distinguish two kinds of HGOS: i) by region of production (territorial distribution); ii) by region of residence (destination). The difference between these two aggregates will define net importer and exporters of imputed rent $HGOS_r^{prod} - HGOS_r^{res} = nHGOS_r$. As expected, the regional unbalances are strictly related with those of tourism, so the deficit/surplus has then been distributed across regions following this procedure:

if: $nHGOS_r > 0$ (net exporter)

[39] $$\tilde{t}_{rs;cpa54:L} = nHGOS_{r;cpa54:L} \cdot \frac{ins_{sr}}{ins_{.r}} \qquad \forall r \neq s$$

if: $nHGOS_r < 0$(net importer)

[40] $$\begin{cases} \tilde{t}_{rr;cpa54:84} = xpb_{r;cpa54:84} & (a) \\ \tilde{t}_{sr;cpa54:L} = nHGOS_{s;cpa54:L} \cdot \frac{ins_{rs}}{ins_{.s}} \qquad \forall r \neq s & (b) \end{cases}$$

where:
ins = night spent by Italians nonresident in the region

- Electricity, gas, steam and air-conditioning (cpa54:D): electricity

In section 2.1.2 we discussed how REB have been use for building the columns of costs of the Use matrix for sector rr43:D and product cpa54:D. Same information could be utilized for estimating the trade matrices of the cpa54:D product. Given the surplus/deficit in the intermediary and final trade of electricity provided by the REBs (nEl$_r$) the multiregional trade flows matrix has been built through the following formulas:

if: $nElx_r > 0$ (net intermediate exporter)

[41] $$\widetilde{tx}_{rs;cpa54:D} = pun \cdot nElx_{r;cpa54:D} \cdot \frac{xpb_s}{xpb} \qquad \nabla r \neq s$$

if: $nElx_f > 0$ (net final exporter)

[42] $$\widetilde{tf}_{rs;cpa54:D} = pun \cdot nElf_{r;cpa54:D} \cdot \frac{twh_s}{twh} \qquad \nabla r \neq s$$

if: $nElx_r < 0$(net intermediate importer)

[43] $$\begin{cases} \widetilde{tx}_{rr;cpa54:D} = pun \cdot twx_r & (a) \\ \widetilde{tx}_{sr;cpa54:L} = pun \cdot nElx_{s;cpa54:D} \cdot \frac{xpb_r}{xpb} \quad \nabla r \neq s & (b) \end{cases}$$

if: $nElf_r < 0$(net final importer)

[44] $$\begin{cases} \widetilde{tf}_{rr;cpa54:D} = pun \cdot twh_r & (a) \\ \widetilde{tf}_{sr;cpa54:L} = pun \cdot nElf_{s;cpa54:D} \cdot \frac{twh_r}{twh} \quad \nabla r \neq s & (b) \end{cases}$$

where:
pun = National electricity single price per TWh
twh = tera-watt consumed for housing purposes
twx = tera-watt consumed for productive purposes

2.4 INITIAL ESTIMATES: non-BENCHMARK YEARs

For the non-benchmark year, the procedure balanced utilizes $(t-1)$ coefficients. The initial estimate of supply and Use matrices have been performed through the balanced **B** and **C** coefficients at time $(t-1)$, as vapb and xpb are estimated as in subsection. So, for the r-nth region at time (t) the initial Use matrix is obtained through:

[45] $$\widetilde{\mathbf{U}}_{r,(t)} = \mathbf{B}_{r,(t-1)} \cdot \widehat{\widetilde{\mathbf{xpb}}}_{r,(t)}$$

As the Make matrix is the result of the following multiplication:

[46] $$\widetilde{\mathbf{S}}_{r,(t)} = \widehat{\widetilde{\mathbf{xpb}}}_{r,(t)} \cdot \mathbf{C}_{r,(t-1)}$$

Same for the bridge matrices of household expenditure, public administration expenditure and gross fixed investments. The incidence of expenditure by demand function/asset and product at time t have been utilized for spreading the RegAcc available at time t (COICOP-12, COFOG-10, Gross investments by owner sector) over the cpa54 product and services.

This new initial estimate will have, as variance, the ex-post variance resulting from the balancing process at time $(t-1)$.

3. BALANCING THE INITIAL ESTIMATES

3.1 CONSTRAINTS

We could distinguish three kinds of constraints: *i*) internal, *ii*) from RegAcc, *iii*) national SUT. Internal constraints are derived from the accounting framework, so for instance total product supply of a regional must be equal to total product demand, total interregional product export must be equal to total interregional product import. The second type of constraints are from the availability of a series of data provided by official statistical sources at regional level. This means above the regional accounts. At the present ISTAT is providing the following data: i) Value added at basic prices rr28, ii) Total net indirect taxes on products, iii) household expenditure at market prices COICOP-12, iv) PA expenditure at market prices COFOG-10, v) total NPIHS, vi) total AIDV plus Changes on inventories, vii) Net total import. Third type of constraints relies on the national SUT.

3.2 BALANCING SYSTEM OF ACCOUNTING IDENTITIES

The SB balancing of the MRSUT of both each single regional SUTs and the multiregional trade matrices (intermediate and final) **T,** has been performed simultaneously, through the following system of balancing identities:

$$[47]\quad \begin{cases} \tilde{\mathbf{S}} \cdot i + \check{\check{\tilde{\mathbf{T}}}}_x \cdot i + \check{\check{\tilde{\mathbf{T}}}}_f \cdot i + \widetilde{\mathbf{mwi}} + \widetilde{\mathbf{mwf}} \equiv \widetilde{\mathbf{U}} \cdot i + \tilde{\mathbf{F}} \cdot i + \check{\check{\tilde{\mathbf{T}}}}_x \cdot i + \check{\check{\tilde{\mathbf{T}}}}_f \cdot i + \widetilde{\mathbf{ew}} & (a) \\ \check{\mathbf{T}}_x \cdot i + \check{\mathbf{T}}_f \cdot i \equiv \check{\mathbf{T}}_x \cdot i + \check{\mathbf{T}}_f \cdot i & (b) \\ i \cdot \tilde{\mathbf{S}} \equiv i \cdot \widetilde{\mathbf{U}} + i \cdot \widetilde{\mathbf{Y}} & (c) \\ \overline{\mathbf{Y}} \equiv \widetilde{\mathbf{Y}} \cdot \mathbf{G}_y & (d) \\ \overline{\mathbf{F}} \equiv \tilde{\mathbf{F}}' \cdot i & (e) \\ \mathbf{nm} \equiv \left[\left(\check{\mathbf{T}}_x \cdot i + \check{\mathbf{T}}_f \cdot i\right) - \left(\check{\mathbf{T}}_x \cdot i + \check{\mathbf{T}}_f \cdot i\right)\right] \cdot i + [(\mathbf{mwi} + \mathbf{mwf}) - (\mathbf{ewi} + \mathbf{ewf})] \cdot i & (f) \\ \begin{bmatrix} 0 & \mathbf{U}_{ita} & \mathbf{F}_{ita} & \mathbf{ew}_{ita} \\ \mathbf{S}_{ita} & 0 & 0 & 0 \\ 0 & \mathbf{Y}_{ita} & 0 & 0 \\ \mathbf{mwi}_{ita} & 0 & 0 & 0 \\ \mathbf{mwf}_{ita} & 0 & 0 & 0 \end{bmatrix} \equiv \sum_{j=1}^{k} \begin{bmatrix} 0 & \mathbf{U}_j & \mathbf{F}_j & \mathbf{ew}_j \\ \mathbf{S}_j & 0 & 0 & 0 \\ 0 & \mathbf{Y}_j & 0 & 0 \\ \mathbf{mwi}_j & 0 & 0 & 0 \\ \mathbf{mwf}_j & 0 & 0 & 0 \end{bmatrix} & (g) \end{cases}$$

where:
k = # NUTS2 regions,
m= # rr43 sectors,
m*= # rr28 sectors,
n= # cpa54 products,
q = # domestic final demand components,
p= # value added components
S = blocks-diagonal regional Make matrices $[(k \cdot n) \times (k \cdot m)]$;
i = column vector
T = block multiregional trade matrix $[(k \cdot n) \times (k \cdot n)]$ for intermediate (subscript x) and final (subscript f) where diagonals of each single trade matrix is set to 0
mwi/mwf = vectors of intermediate/finale foreign import products $(k \cdot n)$;
U = blocks-diagonal regional Use matrices $[(k \cdot n) \times (k \cdot m)]$;
$\overline{\mathbf{F}}$= Regional domestic final demand components constraints $(k \cdot q)$ by RegAcc;
F = blocks-diagonal regional domestic final demand matrices $[(k \cdot n) \times (k \cdot q)]$;
ew = vector of foreign export products $(k \cdot n)$;
$\overline{\mathbf{Y}}$= blocks-diagonal regional primary input components constraints $[(k \cdot p) \times (k \cdot m^*)]$ by RegAcc;
Y = blocks-diagonal regional primary input components $[(k \cdot p) \times (k \cdot m)]$;
$\mathbf{G}_y$ = aggregation matrix from m rr43 to m* rr28 supplied by regional accounts $[(k \cdot m) \times (k \cdot m^*)]$;
nm = Regional Total Net Import provided by RegAcc (k) .

3.3 The variance-covariance matrix of the initial estimates

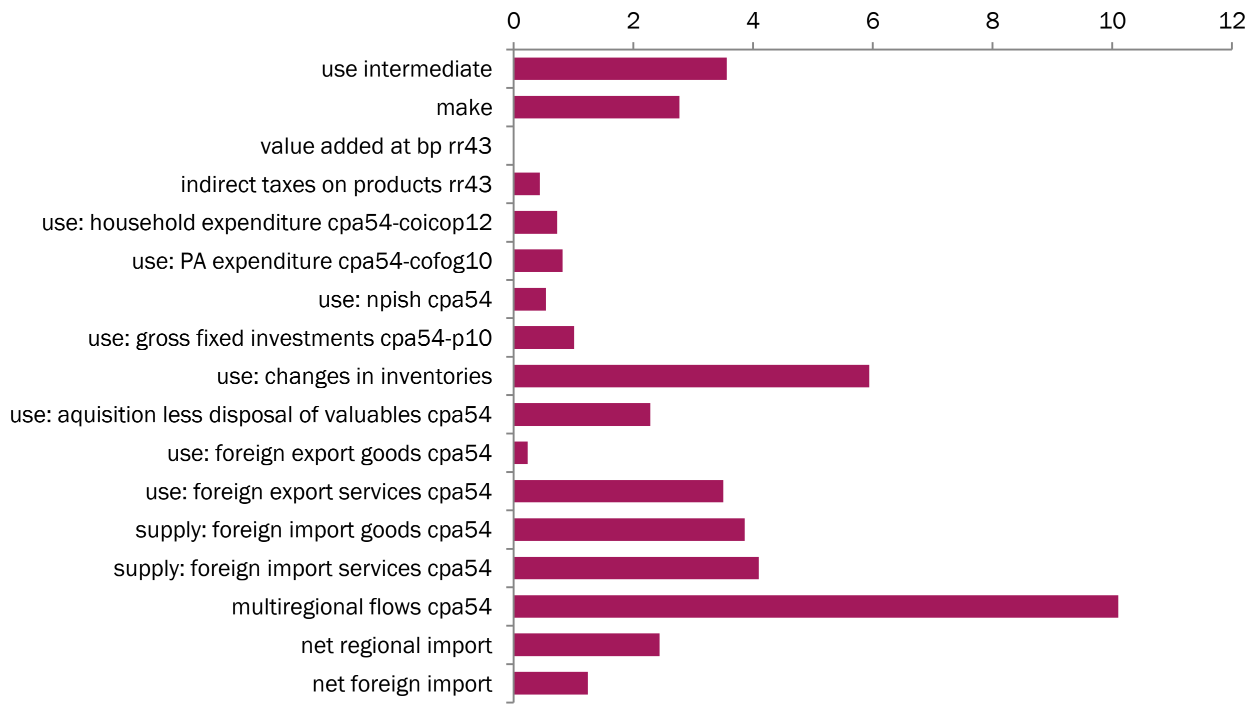
Building the matrix of Var-Cov has had to tackle two kinds of problem. The first concerns the shortage of information on relative reliability and on standard deviation of the estimates. The second concerns the procedure of construction of some initial data which cannot be considered independent, as usually assumed, because they are built based on other initial estimates. An obligatory step in the determination of the matrix Var-Cov has been the tracing of an assignment paradigm of reliability, based on the known economic regional specificity's, numerical and constructive characteristics of the initial data. The model of reliability assignment tries to considerate the factors that could describe the precision of the initial estimate. Once identified, they have been properly combined to determine the reliability. The guidelines of the reliability's assignment have therefore led to a mixed subjective-objective technique. The reliabilities have been distributed in a cardinal way according to two different dimensions, that is: by accounts and by regions and then transformed in variance according to the following equation:

[48] $$\sigma_{rij} = \left(\eta_{ij} \cdot \eta_r^* \cdot \phi_{r,ij}(0)\right)$$

where:
η_{ij}= account unreliability (e.g. use matrix) (min =0)
η_r^* = regional unreliability factor (min = 1)

The range of unreliability by accounts varies from 0, highest/lowest reliability/unreliability, and so 0 variance, to 1.5 (lowest/highest reliability/unreliability). Maximum reliability has been assigned to national SUT values and all regional variables provided by ISTAT Regional Accounts, that is: *i*) Value Added at basic prices rr28, *ii*) Household expenditure COICOP-12, *iii*) PA expenditure COFOG-10, *iv*) total NPIHS, *v*) total inventory and acquisition less disposal of values, *v*) total net indirect taxes on products, *vi*) total net import, *vii*) foreign export of goods by cpa54. Respect to the previous paper[29], using T-FRAME, has allowed to lift reliabilities of: rr43 value added and output, Use and Make values and Changes in inventories. The next figure shows some unweighted average unreliabilities by accounts.

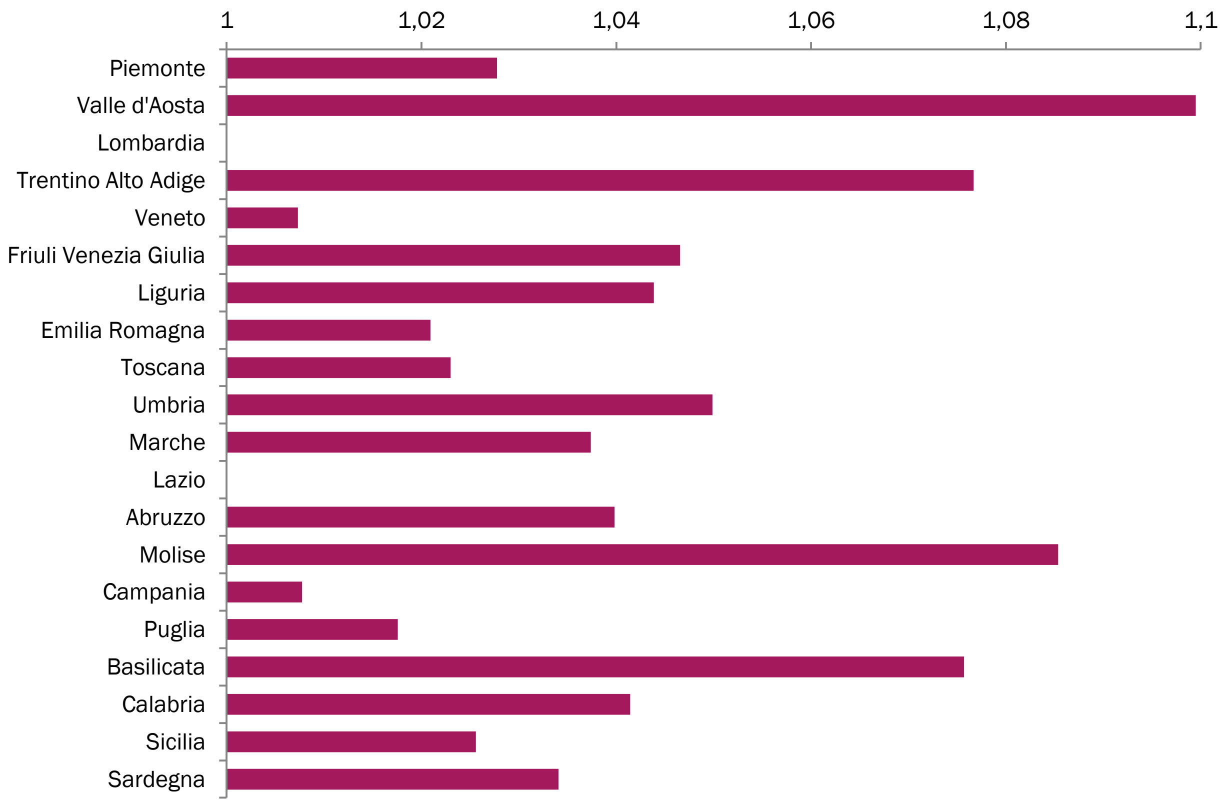
Figure 2.
UNWEIGHTED UNRELIABILITY BY ACCOUNT



[29] Paniccià and Rosignoli (2018).

Figure 3 shows the region-specific factors of accounts unreliability. A proxy of this latter type of unreliability has been provided by the relative regional percentage error of the LFS.

Figure 3.
REGION SPECIFIC UNRELIABILITY FACTOR



source: author calculation from ISTAT Labour Force Survey

3.4 BALANCING RESULTS

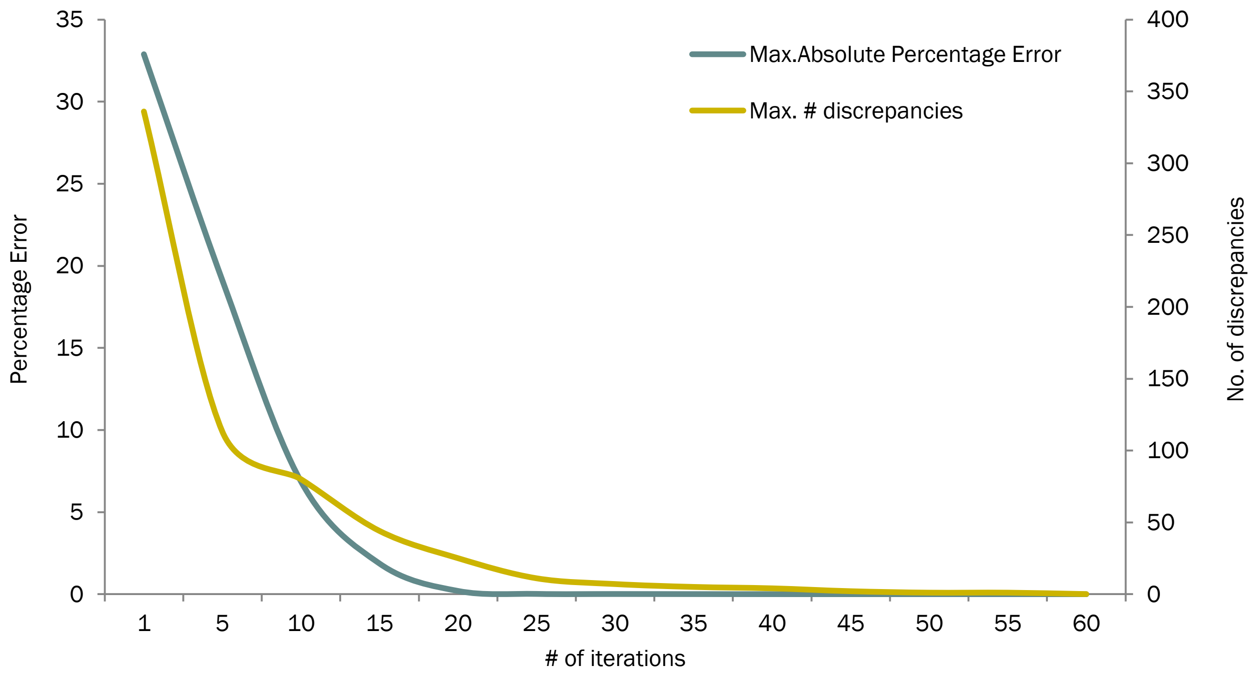
In this subsection two important pieces of information about the balancing process for the benchmark year will be shown. First, the mean absolute percentage adjustment by MRSUT accounts, as in Table 2.

Table 2.
MAPA BY ACCOUNT (MIN, MAX, MEAN)

	Min.	Max.	Mean
use intermediate	0,00	0,10	0,08
make	0,00	0,10	0,05
value added at bp rr43	0,00	0,01	0,01
indirect taxes on products rr43	0,01	0,02	0,01
use: household expenditure cpa54-coicop12	0,00	0,10	0,06
use: PA expenditure cpa54-cofog10	0,10	0,10	0,10
use: npish cpa54	0,10	0,10	0,10
use: gross fixed investments cpa54-p10	0,00	0,10	0,08
use: changes in inventories	0,20	0,30	0,20
use: acquisition less disposal of valuables cpa54	0,10	0,10	0,10
use: foreign export goods cpa54	0,00	0,00	0,00
use: foreign export services cpa54	0,10	0,30	0,17
supply: foreign import goods cpa54	0,10	0,50	0,30
supply: foreign import services cpa54	0,20	0,50	0,32
multiregional flows cpa54	1,00	1,20	1,18
net regional import	1,00	1,00	1,00
net foreign import	0,05	0,20	0,16

Second how the conjugate gradient algorithm has worked for reaching the convergence as shown in Figure 4, where besides the overall mean absolute percentage adjustment (MAPA) by iteration, the max number of discrepancies are displayed.

Figure 4.
BALANCING PROCESS INDICATORS



The resulting balanced MRSUT could be used for the usual I-O analysis as it is (SUT based model) or transformed in a MRIO table by simmetrization through industry technology (Appendix 1).
Just some aggregated figures from the balanced MRSUT. In table 3 the aggregated net resources and domestic uses accounts by nuts2 regions:

Table 3.
REGIONAL NET RESOURCES AND DOMESTIC FINAL USES: 2019 AT CURRENT PRICES (MEURO)

	GDP	Net Import		Net Resources	Household domestic expenditure	PA domestic expenditure	NPISH	Gross Fixed Investments	Changes in Inventories and Valuables	Domestic Final Demand
		Interregional	Foreign							
Piemonte	137941.7	6349.5	-2118.3	242494.5	88695.0	23790.8	700.0	28615.7	371.2	242494.4
Valle d'Aosta	4868.5	371.1	286.5	8235.9	3229.8	1304.6	31.6	977.0	-16.5	8236.3
Lombardia	399793.3	-31133.5	-33212.7	613842.3	210653.4	51324.2	1863.0	70970.4	638.7	613845.0
Trentino-Alto Adige	47310.8	-877.3	-593.8	72223.9	25516.8	9126.9	372.8	10604.6	218.8	72224.0
Veneto	166519.5	6276.3	-17241.3	281904.2	96265.8	25469.0	697.4	32913.6	209.3	281904.9
Friuli-Venezia Giulia	39306.1	3880.1	-3792.7	71461.5	23795.3	7376.1	156.9	7504.3	561.3	71461.7
Liguria	50236.9	1315.2	-193.0	81642.6	32222.3	8987.1	242.3	9920.9	-13.6	81642.5
Emilia Romagna	163052.3	4917.7	-17488.2	275044.0	94617.5	24147.1	788.2	31055.9	-126.1	275044.8
Toscana	122179.1	3207.2	-8268.6	198687.7	75217.9	20834.7	638.9	19624.1	801.8	198687.4
Umbria	22978.9	207.4	1387.5	39947.6	15343.6	5028.5	94.3	4100.9	6.5	39947.6
Marche	42666.1	1220.2	-417.5	74085.3	27033.0	8375.4	211.1	7160.1	689.2	74085.2
Lazio	201986.6	-26652.5	688.9	280610.6	109529.3	31444.3	2217.2	33478.4	-645.5	280611.2
Abruzzo	32936.2	1302.8	718.0	57140.9	20952.0	7308.2	113.3	6476.4	107.0	57140.9
Molise	6487.2	313.0	1072.8	11968.1	4634.2	1938.7	22.6	1223.8	53.8	11968.1
Campania	111065.0	3907.7	9301.2	169537.2	74089.4	30666.9	375.7	18405.4	737.0	169537.8
Puglia	75832.3	4233.0	8675.3	123176.8	54303.2	21174.9	356.3	12415.1	491.0	123176.7
Basilicata	12672.4	1231.5	98.9	23516.5	7955.9	3336.2	54.9	2690.0	-35.2	23515.5
Calabria	33337.4	6702.9	5504.3	54321.3	27966.2	12495.8	149.2	4796.3	136.6	54320.7
Sicilia	89242.4	11516.3	13542.7	147658.1	69857.0	29933.0	469.0	13803.2	237.3	147656.3
Sardegna	35127.0	2207.1	4870.1	60917.2	25502.2	10449.5	227.6	5955.7	68.2	60916.1
Extra	1108.6	-495.7	-93.7	2588.4	0.0	0.0	0.0	512.1	7.1	2588.4
ITALY	1796648.6	0.0	-37273.7	2891004.6	1087379.7	334512.0	9782.2	323204.0	4498.1	2891005.6

ANNEX 1: FROM MRSUT TO MRIO

Despite using a SUT based model could be preferable, many applications and model linkages (see for instance ICIO-OECD tables) require using MRIO table. The debate on the different hypotheses pf simmetrization is well beyond the scope of this report, what it is important to note is that the IRPET methodology form moving from MRSUT to MRIO is based on the industry technology as defined in Miller and Blair (2009)[30]. Moreover, the methodology for simmetrization used is heavily based on the approach proposed by Timmer *et alii* (2012)[31].
In that approach each single regional Use is equivalent to the sum of the imported Use flows plus the intraregional Use. In formal terms for the intermediary flows:

[1.1a] $U_r^x = \sum_{s=1}^{m} U_{sr}^x$

For final flows:

[1.1b] $U_r^f = \sum_{s=1}^{m} U_{sr}^f$

That is Use (final/intermediary) of region r-nth is the sum of imported Use flows from the s-nth regions (included r itself). From the [1.1a] it is straightforward to derive the symmetric matrix Z under industry technology condition, that ais:

[1.2a] $Z_r = \sum_{s=1}^{m} D_s \cdot U_{sr}^x$

And the sectoral final flows

[1.2b] $F_r = \sum_{s=1}^{m} D_s \cdot U_{sr}^f$

Where: D_s is the market share matrix of the s-*nth* exporting region

Of course, value added, and net indirect taxes do not need simmetrization.
If we build a block matrix **U** properly composed by origin and destination region by U_{sr}^x Use matrices, and a block diagonal matrix **D** made up on the diagonal by regional market shares the block matrix **Z** containing the symmetric interregional intermediate sectoral flows (Z_{sr}) is the result of the following multiplication:

[1.3a] $\mathbf{Z} = \widehat{\mathbf{D}} \cdot \mathbf{U}$

Same as for domestic final demand:

[1.3b] $\mathbf{F} = \widehat{\mathbf{D}} \cdot \mathbf{UF}$

where **F** contains the interregional final sectoral flows (F_{sr})

Nonetheless some adjustments should be made to this approach to use it for symmetrize our MRSUT that is:

1) In the MRSUT interregional flows both intermediate and final are Chenery type.
2) In the MRSUT foreign imported flows are treated separately and do not have country breakdown.
3) MRIO table must be constrained to the national IOT.

To adjust for condition 1) and 2) the [1.2a] will become:

[1.4a] $Z_r = \sum_{s=1}^{m} D_s \cdot \left(\widehat{Rx}_{sr} \cdot U_r^x\right) + D \cdot \left(\widehat{MWx}_r \cdot U_r^x\right)$

---

[30] Miller, Ronald E., and Peter D. Blair. *Input-output analysis: foundations and extensions*. Cambridge university press, 2009.
[31] Timmer, Marcel, et al. *The world input-output database (WIOD): contents, sources and methods*. No. 20120401. Institute for International and Development Economics, 2012.

where:
$\widehat{Rx}_{sr}$ = multiregional intermediate import coefficients from region s-*nth* to region r-*nth*
$\widehat{MWx}_{r}$ = foreign intermediate import coefficients from abroad to region r-*nth*
D = national market shares

And the [1.2b] domestic final demand will become:

[1.4b] $F_r = \sum_{s=1}^{m} D_s \cdot \left(\widehat{Rf}_{sr} \cdot U_r^f\right) + D \cdot \left(\widehat{MWf}_r \cdot U_r^f\right)$

where:
$\widehat{Rf}_{sr}$ = multiregional final import coefficients from region s-*nth* to region r-*nth*
$\widehat{MWf}_{r}$ = foreign final import coefficients from abroad to region r-*nth.*

Sectoral foreign export will be:

[1.4c] $EW_r = D_r \cdot U_r^{ew}$

In terms of block matrices [1.3a] will become:

[1.5a] $\mathbf{Z} = \widehat{\mathbf{D}} \cdot \left(\mathbf{Rx} \cdot \widehat{\mathbf{U}}\right) + \widehat{D} \cdot \left(\widehat{\mathbf{MWx}} \cdot \widehat{\mathbf{U}}\right)$

where:
$\widehat{\mathbf{U}}$ = block diagonal matrix of regional Use total intermediate flows.
$\widehat{D}$ = block diagonal matrix of national market shares.
As for domestic final demand:

[1.5b] $\mathbf{F} = \widehat{\mathbf{D}} \cdot \left(\mathbf{Rf} \cdot \widehat{\mathbf{UF}}\right) + \widehat{D} \cdot \left(\widehat{\mathbf{MWf}} \cdot \widehat{\mathbf{UF}}\right)$

The resulting MRIO table from [15] lacks consistency with the official IOT ind. by ind., under industry technology, released by ISTAT. Despite the initial MRSUT is fully consistent with the national SUT the MRIO table does not fully comply with the national IOT constraint. The reason relies in the starting symmetrized national SUT which is more disaggregated than the published ones[32]. Even if discrepancies are relatively small (max 5%), to ensure also national consistency with official IOT, the initial MRIO tables are inserted in an adjusted rAs3D which preserves all sectors specific figures (e.g. Value added, net indirect taxes) and regional account constraints.
To be remarked that adjustments made by the rAs3D procedure results in very small adjustment of initial MRIO figures.

[32] The most disaggregated national SUT, estimated but not released by ISTAT, is made up by 262 CPA products and 98 NACE. Rev.2 sectors.

ANNEX 2: FROM MRSUT TO IRSUT

Input-Output datasets are now providing both Interregional SUT and Interregional symmetric matrices. Indeed, the interregional SUT are not fully *Isard type* complying, they are very often the result of an Isardization[33] of a multiregional SUT (see for instance FIGARO tables).
For doing the same operation on estimated MRSUT it is important to note that multiregional trade flows are divided in two categories: intermediate and final.
Starting with intermediate flows. We define a generic element *qx*:

[2.1] $qx_{ij}^{r} = \frac{u_{ij}^{r}}{\sum_{j=1}^{m} u_{ij}^{r}} =$

That is the share of total intermediate demand of product i-*nth* demanded by sector *j-nth* in region *r-nth*.
For any region *r-nth* we could define a matrix $\mathbf{Qx}_r$ with the same dimensions of the intermediate Use matrix.
The block diagonal matrix **Qx** will be composed by the regional ***Qx*** shares.
By rearranging the multiregional bilateral intermediate flows we could built a matrix **Tx** composed by diagonal sub matrices $\mathbf{Tx}_{rs}$ that is: the intermediate product flows between region r (origin) and region s (destination). Given these matrices it is quite straightforward to produce the isardization of intermediate Use trough:

[2.2] $\mathbf{U}^{\mathbf{interreg}} = \mathbf{Tx} \cdot \widehat{\mathbf{Qx}}$

Mutatis mutandis it could be possible to redo the same procedure for the final part of multiregional Use. In this case the generic share *qf* is computed over the domestic final demand components (*k-nth*).

[2.3] $qf_{ik}^{r} = \frac{u_{ik}^{r}}{\sum_{k=1}^{m} u_{ik}^{r}}$

The **Tf** matrix will the result of the rearrangement of the final multiregional flows as done previously. The Isardization of final Use will be:

[2.4] $\mathbf{U}^{\mathbf{final}} = \mathbf{Tf} \cdot \widehat{\mathbf{Qf}}$

[33] I apologize with the reader for this term but I did not find a more proper definition

ANNEX 3: FROM REGIONAL IOT TO MULTI-LMA TABLE FOR TUSCANY

The role played by the different local areas in determining the economic structure of Tuscany has been well recognized. ISTAT has introduced and identified in the early 80s a typology of local areas that is the Labour Market Area (LMA). LMA is also a Eurostat geographical unit and can be defined as: an economically integrated area within which residents can find jobs within a reasonable commuting distance or can change their employment without changing their place of residence.

Why using LMAs for analyzing the economic structure of Tuscany. Indeed, although Tuscany is a small/medium region, it is characterized by a significant heterogeneity in terms of economic and social structure. Beside the urban LMAs (mostly Florence and Pisa), there are LMAs identified as industrial districts based on SMEs. We could also find LMAs specialized in pharmaceutical products and mechanical equipment and characterized by large and medium size enterprises. Quite significant is also the number of rural and/or tourist LMAs.

Having say that, it is straightforward, in analyzing the Tuscan economic structure, to detect how and how much the different area (LMA) are linked with the rest of Italy and the rest of the World, and how and how much they interrelate each other, through trade and income flows.

Using a multi LMA I-O model could then allow to overcome the traditional readings by sector and size breakdown, that are no longer sufficient to adequately describe the heterogeneity of the Tuscan socio-economic system and the role played by the LMAs, which are resulting from the endogenous and (largely) autonomous organization of the choices and actions of institutional factors.

To be remarked that, a significant impulse to this analysis has been provided by the availability, in the recent years, of additional data sources at local level, in particular Business Register, FRAME and Structural Business Statistics (already quoted in the main text), which allow to get information by KAU and local KAU. Moreover, IRPET has conducted some surveys for detecting manufacturing trade flows of Tuscan enterprises by region and LMA. These data sources, along with other surveys on tourism flows within Tuscany, have allowed to expand the IRPET multiregional I-O model by splitting Tuscany region in the 49 LMAs (see map in Appendix 2 figure 2.1) and including them in the multiregional model.

However, the first step in building the Multi LMA I-O table (MLIO henceforth) has been the estimate of a more disaggregated I-O table for Tuscany acting as “across LMAs” constraint. The extended granularity is needed to catch the specialization of the LMAs properly. For instance, in the regional IOT extracted from MRIO, a unique sector rr43 combines Textiles, Wearing, Leather, Leather products and Shoes manufacture. Nonetheless the components of the above rr43 sector show significant differences in the intermediate input structure (especially Leather), and, in Tuscany, there are LMAs with a high specific specialization: Prato LMA is specialized in Textile, Florence in Wearing, as S. Miniato in Leather.

In Appendix 2 Table 2.1 the MRIO rr43 and the new granularity associated with the more disaggregated (rr43 extended) Tuscan IOT.

Once defined the regional constraint we could proceed in estimating the MLIO table.

In figure 2.2 Appendix 2 the flow chart of the assembly line is shown. The most time-consuming step has been the construction of each single initial uniLMA IOT. In the same graph it is possible to note the data sources utilized, in which a significant role has been played by the T-Frame. For providing the initial estimates of the regional and multiLMA trade flows we have used a survey conducted by IRPET in 2017 in which, an item was about the geographical destination of turnover has been asked to LKAU of the sample.

The three components of the estimating procedure, that is: initial uniLMA IOTs, trade flows and Regional IOT will then be introduced in the estimating iterative procedure (Figure 2.2).

Unlike the MRSUT, the MLIO table is not simultaneously balanced.

First, each single uniLMA IOT is balanced through SB algorithm complying with both internal (supply and demand) and external constraints (regional IOT). To be note that, at this stage, inter LMA trade is specified, within the uniLMA tables, as sectoral net import.

Second, multiLMA sectoral flows have then balanced, given the initial estimates and the uniLMA IOTs constraints.

Merging the uniLMA IOTs and the sectoral multi LMA trade flows has then allow to build the Multi LMA table, which accounting structure is shown in Figure 2.3 Appendix 2.

thats

APPENDIX 1: CLASSIFICATIONS IN MRSUT

Table 1.1
SECTORS

NACE rev.2 rr43	Description	NACE rev.2 rr28
AA	Agriculture, Hunting, Forestry, Logging And Related Services	AA
AB	Fishing, Aquaculture, Support Services To Fishing	AB
B	Mining And Quarrying	B
CA	Food, Beverages, and Tobacco	CA
CB	Textiles, Wearing Apparel and Leather	CB
CCA	Wood And of Products of Wood And Cork, Except Furniture	CCA.CCB
17	Paper And Paper Products	
18	Printing And Reproduction of Recorded Media	
CD	Coke And Refined Petroleum Products	CD-CE-CF
CE	Chemicals And Chemical Products	
CF	Basic Pharmaceutical Products and Pharmaceutical Preparations	
22	Rubber And Plastic Products	CG (22-23)
23	Other Non-Metallic Mineral Products	
24	Basic Metals	CH
25	Fabricated Metal Products, Except Machinery And Equipment	
CI	Computer, Electronic and Optical Products	CI-CJ-CK
CJ	Electrical Equipment	
CK	Machinery And Equipment N.E.C.	
29	Motor vehicles	CL
30	Other Transport Equipment	
31_32	Manufacturing N.E.C	CM
33	Repair And Installation Of Machinery And Equipment	
D	Electricity, Gas, Steam and Air-Conditioning	D
E	Natural Water; Water Treatment, Sewerage; Waste Collection	E
F	Construction	F
G	Wholesale And Retail Trade; Repair Of Motor Vehicles And Motorcycles	G
H	Transportation And Storage	H
I	Accommodation And Food Service Activities	I
JA	Publishing, Motion Picture, Video, Sound And Broadcasting Activities	JA-JB-JC
JB	Telecommunications Activities	
JC	Computer Programming, Consultancy And Related Activities	
K	Financial And Insurance Activities	K
L	Real Estate Activities	L
MA	Legal And Accounting Consulting, Architectural And Engineering Activities	MA-MB
72	Scientific Research and Development Activities	
73-74-75	Adv. and market research services. Other professional, scientific and technical services	
N	Other Administrative Activities	N
O	Public Administration and Defense; Compulsory Social Security	O
P	Education	P
Q	Human Health and Social Work Activities	Q
R	Arts, Entertainment and Recreation	R
S-T-U	Other Services	S-T-U

Table 1.2
PRODUCTS

CPA 2.1 Code	Description
1	Products of agriculture, hunting and related services
2	Products of forestry, logging and related services
3	Fish and other fishing products; aquaculture products; support services to fishing
B	Mining and quarrying
10-12	Food products, beverages and tobacco products
13-15	Textiles, wearing apparel and leather products
16	Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials
17	Paper and paper products
18	Printing and recording services
19	Coke and refined petroleum products
20	Chemicals and chemical products
21	Basic pharmaceutical products and pharmaceutical preparations
22	Rubber and plastics products
23	Other non-metallic mineral products
24	Basic metals
25	Fabricated metal products, except machinery and equipment
26	Computer, electronic and optical products
27	Electrical equipment
28	Machinery and equipment N.E.C.
29	Motor vehicles, trailers and semi-trailers
30	Other transport equipment
31-32	Furniture; other manufactured goods
33	Repair and installation services of machinery and equipment
D	Electricity, gas, steam, and air-conditioning
36	Natural water; water treatment and supply services activities and other waste management
37-39	Sewerage; waste collection, treatment, and disposal activities
F	Constructions and construction work
45	Wholesale and retail trade and repair services of motor vehicles and motorcycles
46	Wholesale trade services, except of motor vehicles and motorcycles
47	Retail trade services, except of motor vehicles and motorcycles
49	Land transport services and transport services via pipelines
50	Water transport services
51	Air transport services
52	Warehousing and support services for transportation
53	Postal and courier services
I	Accommodation and food services
58	Publishing services
59-60	Motion picture, video and television program production services, sound recording and music publishing
61	Telecommunications services
62-63	Computer programming, consultancy, and related services; information services
64	Financial services, except insurance and pension funding
65	Insurance, reinsurance, and pension funding services, except compulsory social security
66	Services auxiliary to financial services and insurance services
L	Real estate services
69-71	Legal and accounting consulting services and Architectural and engineering services
72	Scientific research and development services
73-75	Advertising and market research services + Other professional, scientific and technical services
77-82	Other administrative activities
84	Public administration and defense services; compulsory social security services
P	Education services
86-88	Human health and social services
90-93	Creative, arts entertainment and cultural services+ sporting and amusement services
94-96	Other personal services
T-U	Services of households as employers; undifferentiated goods and services produced by h. for own use

Table 1.3
HOUSEHOLD EXPENDITURE FUNCTIONS: COICOP-12

COICOP 1999 code	Description
01	Food and non-alcoholic beverages
02	Alcoholic beverages, tobacco
03	Clothing and footwear
04	Housing, water, electricity, gas, other fuels, actual and imputed rent
05	Furnishings, household equipment and routine maintenance of the house
06	Health
07	Transport
08	Communication
09	Recreation and culture
10	Education
11	Restaurants and hotels
12	Miscellaneous goods and services

Table 1.4
PUBLIC ADMINISTRATION EXPENDITURE FUNCTIONS: COFOG-10

COFOG code	Description
01	General public services
02	Defense
03	Public order and safety
04	Economic affairs
05	Environmental protection
06	Housing and community amenities
07	Health
08	Recreation, culture and religion
09	Education
10	Social protection

Table 1.5
GROSS FIXED INVESTMENTS: NON-FINANCIAL ASSETS

Non-Financial Asset code	Description
115	Cultivated biological resources
1131	Transport equipment
11321	Computer hardware
11322	Telecommunications equipment
1139-114	Other machinery and equipment and weapons systems
111-112	Total construction
1171	Research and development
1172-1174-1179	Mineral exploration and evaluation, entertainment, literary or artistic originals
1173	Computer software and databases

Table 1.6
OTHER FINAL DEMAND COMPONENTS

	Description
P.52	Changes in inventories
P.53	Acquisition less disposal of values
S.15	NPISH expenditure
P.6ri	Regional export of goods and services, intermediate
P.6rf	Regional export of goods and services, final
P6w	Foreign export of goods and services

Table 1.7
PRIMARY INPUTS AND IMPORT

	Description
D1-B3	Value Added at basic prices
D21-D31	Net Taxes on products
P.7ri	Regional import of goods and services, intermediate
P.7rf	Regional import of goods and services, final
P.7wi	Foreign import of goods and services, intermediate
P.7wf	Foreign import of goods and services, final

Table 1.8
THE RR63 SECTOR FURTHER DISAGGREGATED IN THE RR63 EXTENDED

rr63	Description	rr63 extended	Description
B	Mining and quarrying	05-06	Extraction of oil, gas and coal
		07-08	Other minerals
		09	Mining services
13-15	Textiles, wearing apparel, leather, and related products	13	Textiles
		14	Wearing apparel
		15.1	Tanning and dressing of leather; luggage, handbags, saddleryand harness; dressing and dyeing of fur
		15.2	Footwear
31-32	Furniture and other manufactured goods	31	Furniture
		32.1	jewelry, bijouterie and related articles
		32	Other manufacture
D	Electricity, gas, steam and air conditioning supply	35.1	Electric power generation, transmission and distribution
		35.2-35.3	Gas, steam, and air conditioning supply

APPENDIX 2: CLASSIFICATIONS IN TUSCAN EXTENDED IOT

Figure 2.1
THE TUSCAN LABOR MARKET AREAS IN THE MULTILMA TABLE

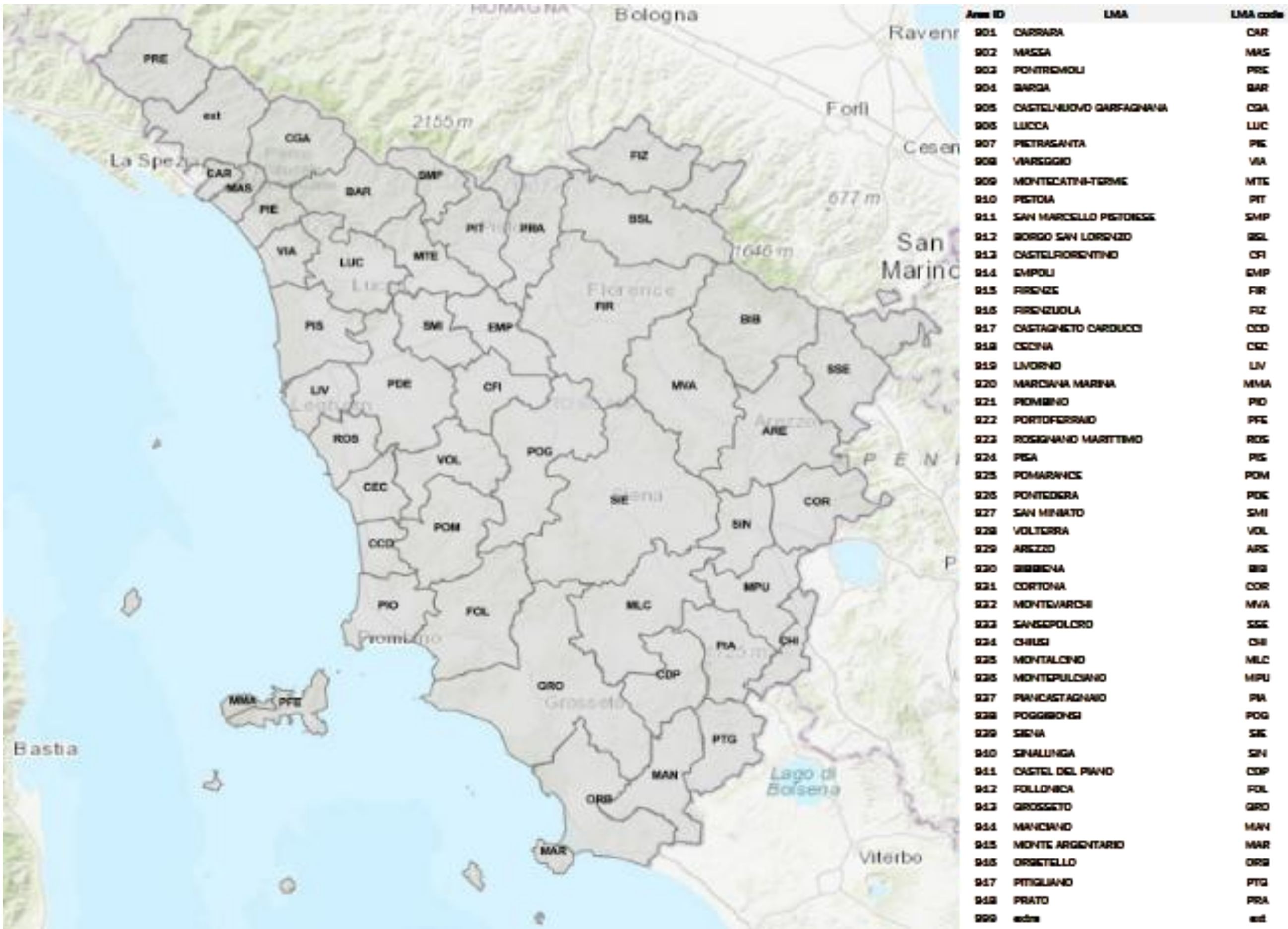


Table 2.1
THE RR43 SECTOR FURTHER DISAGGREGATED IN THE RR43 EXTENDED

rr43	Description	rr43 extended	Description
13-15	Textiles, wearing apparel, leather, and related products	13	Textiles
		14	Wearing apparel
		15.1	Tanning and dressing of leather; luggage, handbags, saddleryand harness; dressing and dyeing of fur
		15.2	Footwear
31-32	Furniture and other manufactured goods	31	Furniture
		32.1	Jewelry, bijouterie, and related articles
		32	Other manufacture
D	Electricity, gas, steam and air conditioning supply	35.11	Electric power generation
		35.12-35.13.35.14	Electric power transmission and distribution
		35.2-35.3	Gas, steam, and air conditioning supply
E	Natural Water; Water Treatment, Sewerage; Waste Collection	36	Natural water, collection, treatment, distribution
		37	Sewerage
		38-39	Waste collection

Figure 2.2
THE MULTILMA TABLE ASSEMPLY LINE

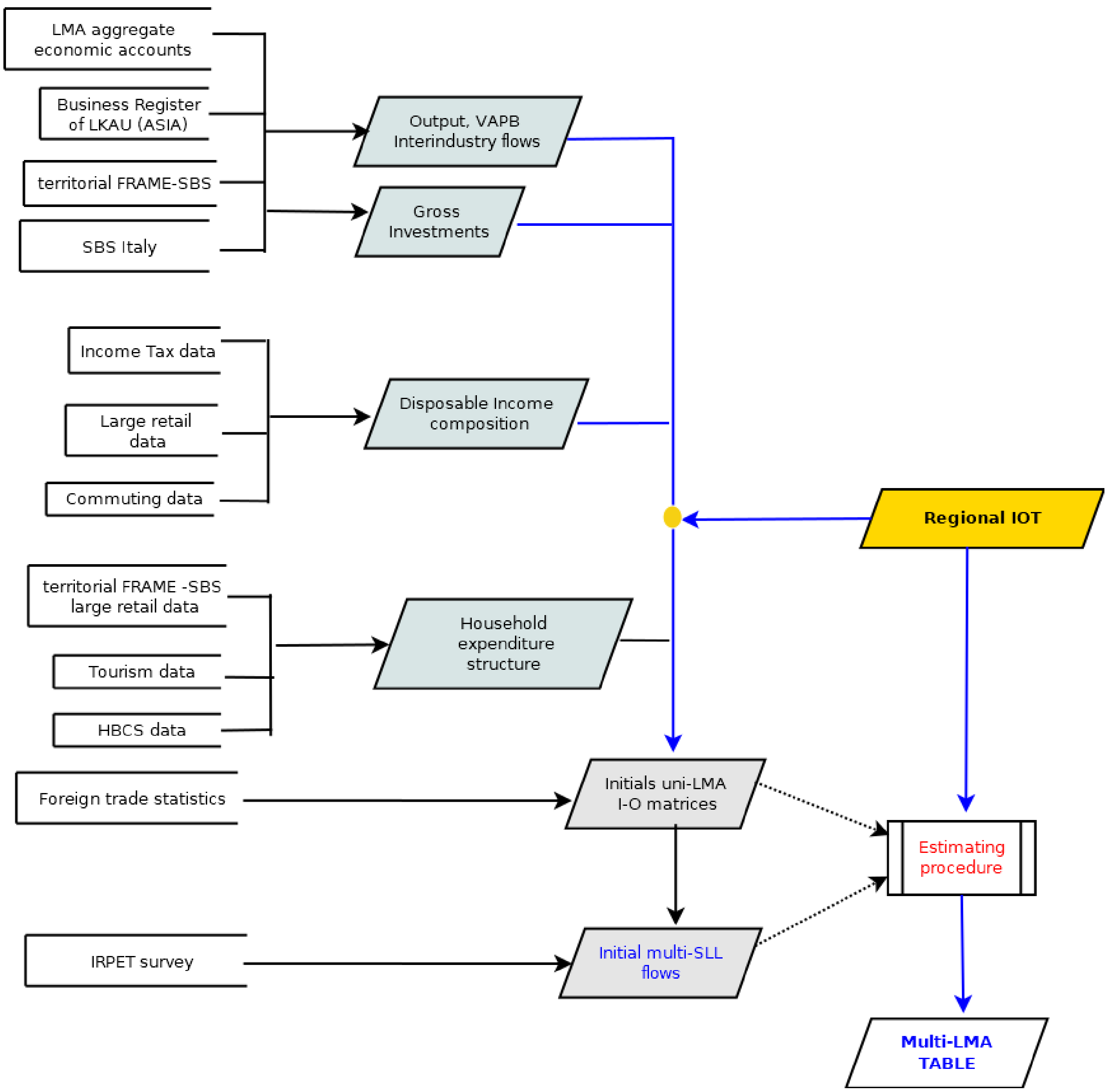


Figure 2.3
THE MULTILMA TABLE ACCOUNTING SCHEME